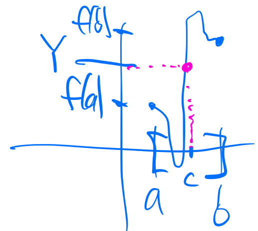
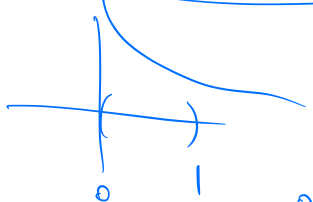
Recall: Def: f is continuous at $x=c$ if: $\lim_{x\to c} f(x)$ exists $= f(c)$.

(i.e. $\lim_{x\to c^+}$ exists & $\lim_{x\to c^-}$ exists & agree)

IVT (Intermediate Value Thm): If f continuous on $[a,b]$, then for all ($\forall$) values of Y between $f(a)$ & $f(b)$, there exists ($\exists$) $c\in[a,b]$ such that $f(c)=Y$

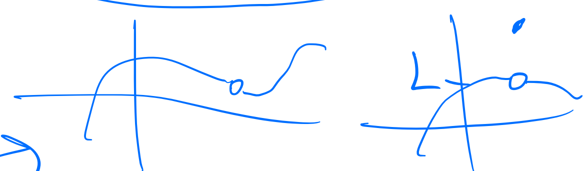




$f(x)=\frac{1}{x}$ is cont. on $(0,1)$

But $f(0)$ DNE.

Use of IVT: finding zeros (if cont function changes sign, it must cross zero.)

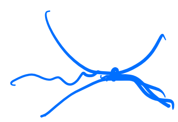


(Subtle aside: If domain of f is $[a,b]$, we say that f is cont. at $x=a$ if $\lim_{x\to a^+} f(x)$ exists & agrees with $f(a)$)

If f is not continuous at $x=c$, but $\lim_{x\to c} f(x) = L$ exists, then we can define the continuous extension:

$$g(x)=\begin{cases} f(x), & x\neq c \\ L, & x=c \end{cases}$$

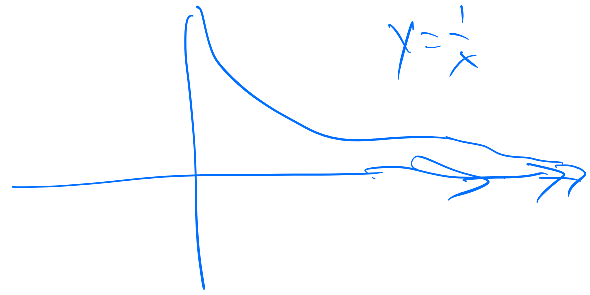
Used Squeeze Thm to give



continuous extension to:

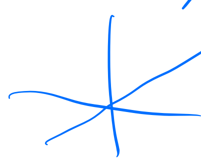
$$g(x)=\begin{cases} \frac{\sin x}{x}, & x\neq 0. \\ 1, & x=0. \end{cases}$$

Limits at ∞ (As opposed to at $x = c$) Really: limit as $x \to \infty$.



E.g.: $\lim_{x \to \infty} \frac{1}{x} = 0$.

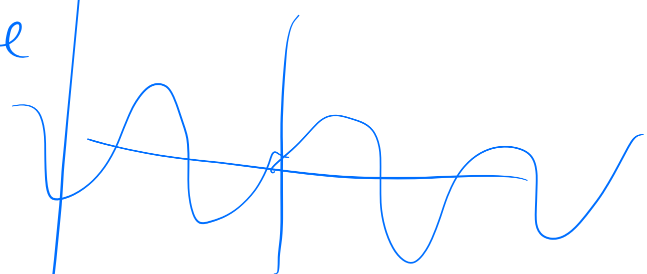
E.g.: $\lim_{x \to \infty} x = \infty$ (Not a real limit)



The limit of $f(x) = x$ as $x \to \infty$ does not exist, which we write as:

$$\lim_{x \to \infty} x = \infty.$$

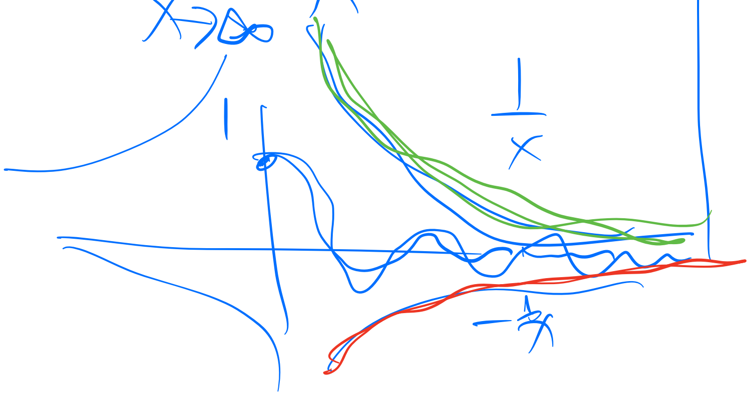
E.g.: $\lim_{x \to \infty} \sin x$ DNE.



E.g.: $\lim_{x \to \infty} \frac{\sin x}{x} = 0$ ← "Squeeze Thm at ∞":

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to \infty} \frac{-1}{x} = 0$



$$-1 \le \sin x \le 1 \quad (x>0)$$

$$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$$

$$\downarrow \qquad \downarrow \qquad \downarrow \text{ as } x\to\infty$$

$$0 \qquad 0 \qquad 0$$

$$0.9999999\ldots = 1$$

$x = 99$, $\frac{x}{x+1} = \frac{99}{100} = 0.99$

$x = 999$, $\quad$ " $= 0.999$.

E.g.: $\lim_{x\to\infty} \frac{x}{x+1} = 1.$

In the limit, the value approaches 1.

$$\frac{x\cdot\frac{1}{x}}{(x+1)\cdot\frac{1}{x}} = \frac{1}{1+\frac{1}{x}} \to 1, \qquad \frac{1}{x}\to 0.$$

E.g. $\lim_{x\to\infty} \frac{(5x^2-3x+17)\frac{1}{x^2}}{(7+3x-4x^2)\frac{1}{x^2}}$

$$= \lim_{x\to\infty} \frac{5-\frac{3}{x}+\frac{17}{x^2}}{\frac{7}{x^2}+\frac{3}{x}-4} = \frac{-5}{4}$$

So $y = -\frac{5}{4}$ is the equation of a line which is a horizontal asymptote.

Quiz: $\lim_{x\to-\infty} \frac{(3x^2-7x^3+5)\frac{1}{x^3}}{(|4x-3x^2+12x^3|)\frac{|x|^3}{x^3|x|^3}} = \lim_{\boxed{x\to-\infty}} \frac{\cancel{\frac{3}{x}}\boxed{-7}+\cancel{\frac{5}{x^3}}}{\left|\frac{4}{x^2}-\frac{3}{x}\boxed{+12}\right|\frac{|x|^3}{x^3}} = +\frac{7}{12}$

Another approach: Just look at: $\lim_{x\to-\infty} \frac{-7x^3 \to +}{|12x^3| \to +}$

E.g.: $\lim_{x\to-\infty} \frac{3x^2-7x^4}{|4x-4x^2+12x^4|}$

$= -\frac{7}{12}$.

E.g.: $\lim_{x\to+\infty} \frac{3x^2-7x^3+5}{|4x-3x^2+12x^3|} = \frac{-7}{12}$

So $f(x) = \frac{3x^2-7x^3+5}{|4x-3x^2+12x^3|}$ has two horizontal asymptotes

One at $y = \frac{7}{12}$

One at $y = -\frac{7}{12}$

E.g.: $\lim_{x\to\infty} \sin\left(\frac{1}{x}\right) = 0$

$\frac{1}{x} \to 0$

Change variable $x \to \infty$ to $t = \frac{1}{x} \to 0$.

E.g.: $\lim_{x\to\infty} x \cdot \sin\left(\frac{1}{x}\right)$, $x \to \infty$, $\sin\left(\frac{1}{x}\right) \to 0$

$= \lim_{t\to 0} \frac{1}{t} \cdot \sin(t)$.

We already showed that

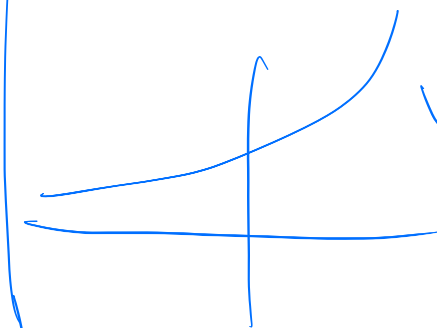
$$\lim_{t\to 0}\frac{\sin t}{t}=1.$$

So

$$\lim_{x\to\infty} x\cdot\sin\left(\frac{1}{x}\right)=1$$

Ex:

$$\lim_{x\to-\infty} e^{\frac{1}{x}}=1 \qquad \frac{1}{x}\to 0$$

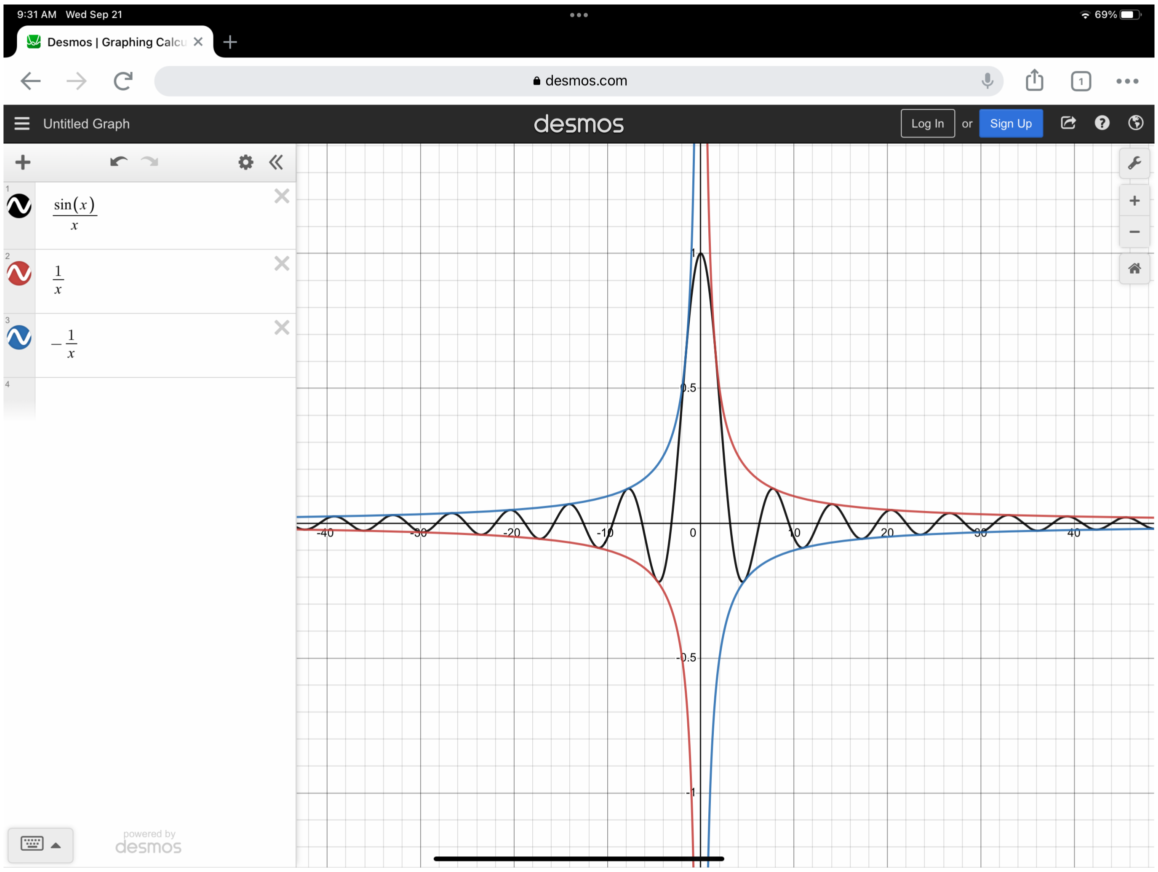


$y=e^x$

Ex:

$$\lim_{x\to 0^-} e^{\frac{1}{x}}=0. \quad = \lim_{t\to-\infty} e^{t}=0.$$

Let $t=\frac{1}{x}\to-\infty$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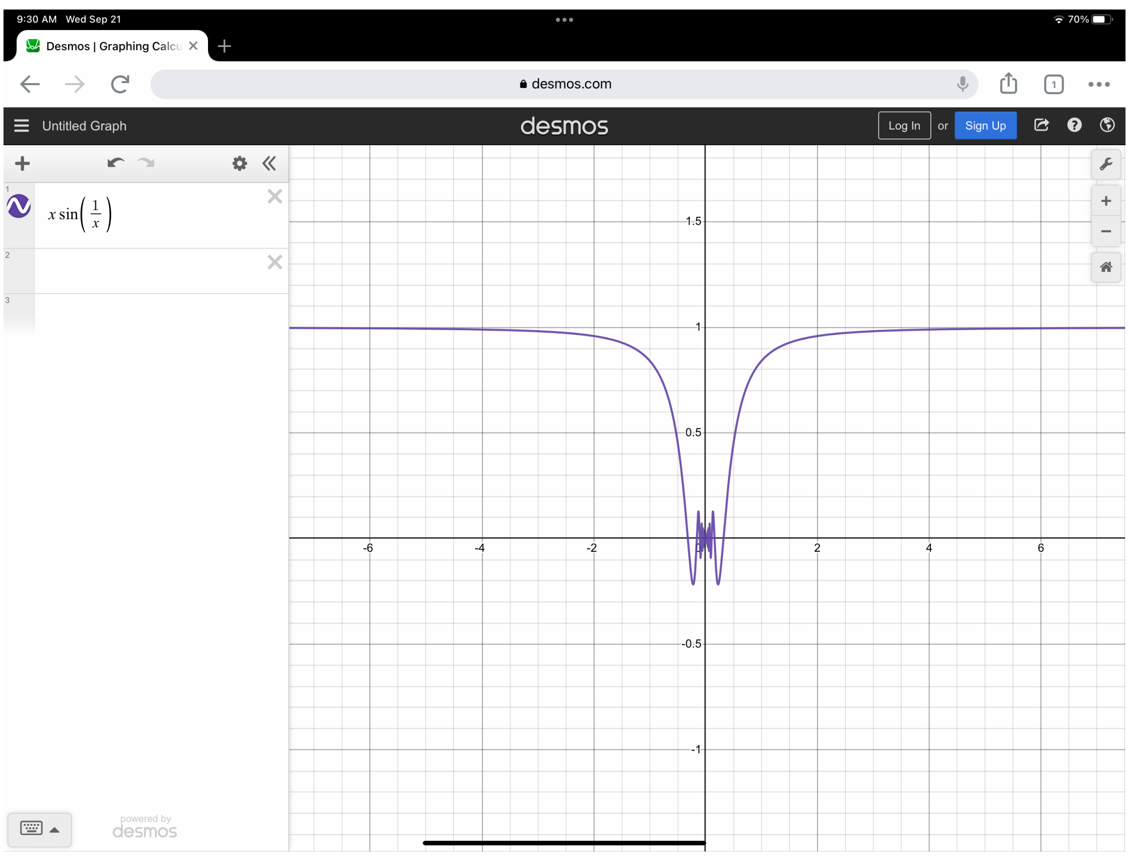
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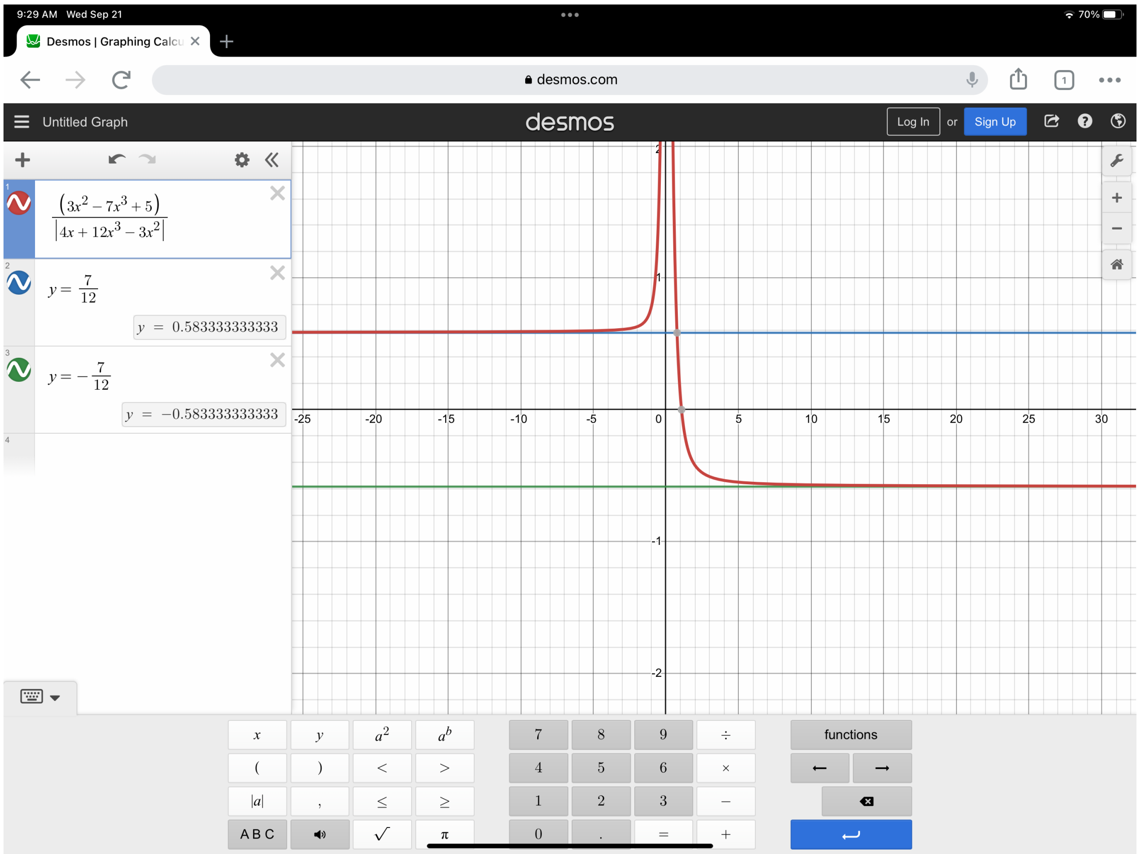
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$\frac{(3x^2-7x^3+5)}{|4x+12x^3-3x^2|}$
$y=\frac{7}{12}$
$y = 0.583333333333$
$y=-\frac{7}{12}$
$y = -0.583333333333$
-25 -20 -15 -10 -5 0 5 10 15 20 25 30
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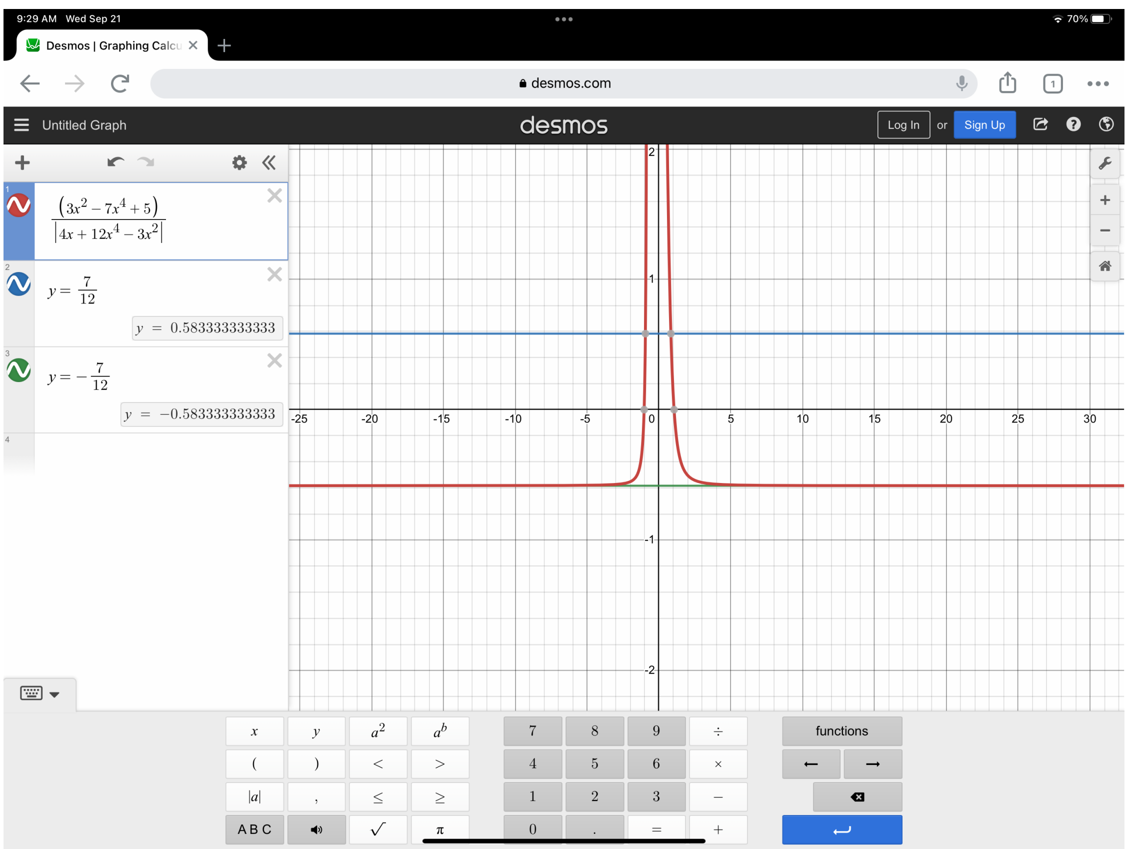
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