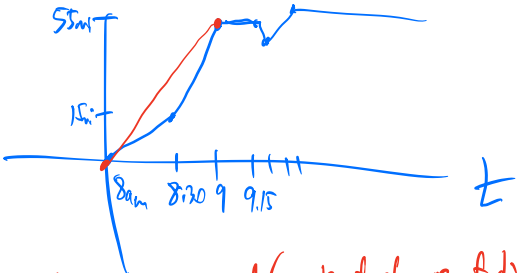
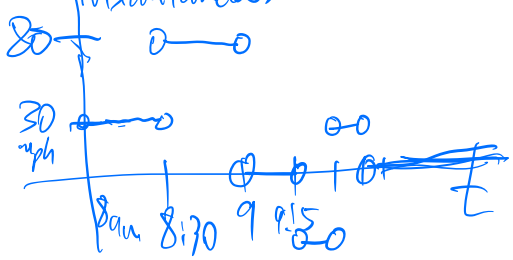
Graph of morning Commute

d = distance from home

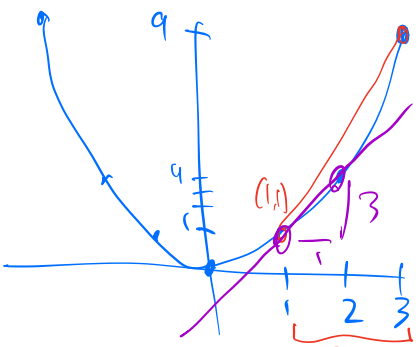


Average speed (= rate of change of distance from home) From 8 to 9 am?

→ Speedometer reading instantaneous



Take a curve $y=x^2$



Average rate of change from $x=1$ to $x=3$? slope $= \frac{8}{2} = 4$.

Equation of line secant to graph of $y=x^2$ at $x=1$ & $x=3$?

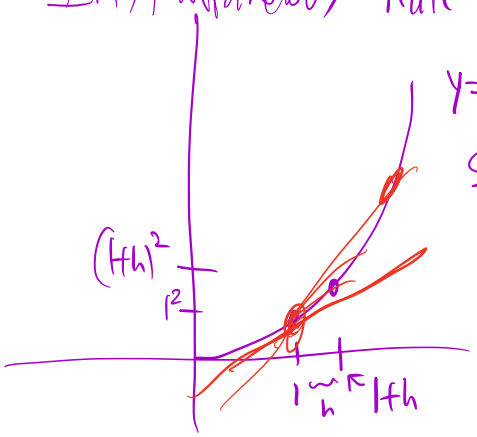
$$y = 4x-3$$

$$y-1 = 4(x-1)$$

Do same for $x=1$ & $x=2$.

$$y-1 = 3(x-1)$$

Instantaneous Rate of Change at $x=1$

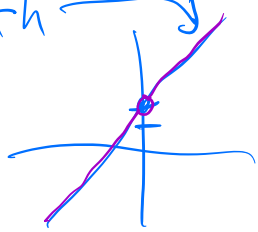


Slope$(h) = \frac{(1+h)^2-1^2}{h}$

plug in $h=0$, Slope$(0) = \frac{(1+0)^2-1^2}{0} = \frac{0}{0}$.

Instead of value at $h=0$, need/want limit as $h \to 0$.

$$\text{Slope}(h) = \frac{(1+h)^2-1^2}{h} = \frac{1+2h+h^2-1}{h} = \frac{2h+h^2}{h} = 2+h$$



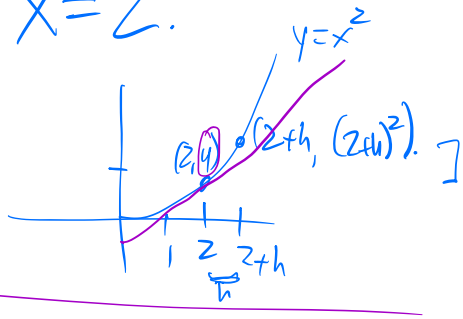
Slope as $h \to 0 \to 2$

2 = instantaneous rate
= slope

$\Rightarrow$ Equation of line tangent to $y = x^2$ at $x = 1$

is: $y - 1 = 2(x-1)$

Quiz: Work out equation of tangent line to $y = x^2$ at $x = 2$.



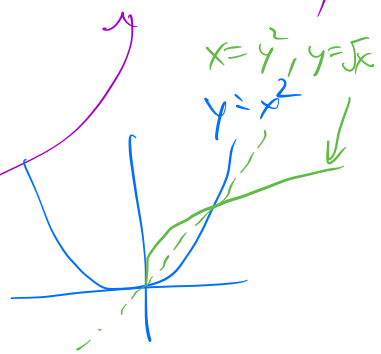
$$\text{Slope}(h) = \frac{(2+h)^2 - 2^2}{h}$$

$$= \frac{4 + 4h + h^2 - 4}{h} = 4 + h$$

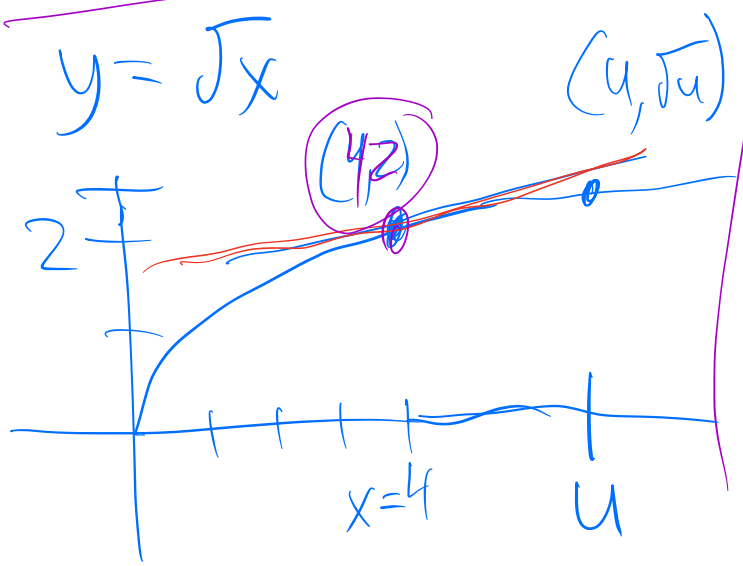
$\to 4$ as $h \to 0$

i.e. $h \to 0$

$y - 4 = 4(x-2)$



$y = \sqrt{x}$



$$\text{Slope}(u) = \frac{\sqrt{u} - 2}{u - 4}$$

Slope of tangent line is $\lim_{u \to 4} \frac{\sqrt{u} - 2}{u - 4}$.

$$\lim_{u\to 4}\frac{(\sqrt{u}-2)(\sqrt{u}+2)}{(u-4)(\sqrt{u}+2)}$$

$$= \frac{u-4}{(u-4)(\sqrt{u}+2)} = \frac{1}{\sqrt{u}+2} \quad \text{if } u\neq 4.$$

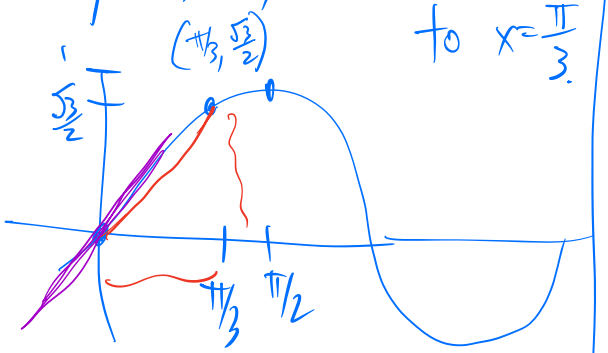
As $u\to 4$,

$$\frac{1}{\sqrt{u}+2} \to \frac{1}{\sqrt{4}+2} = \frac{1}{4}.$$

Eqn:

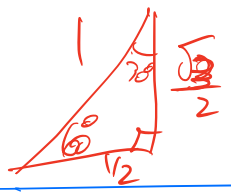
$$y-2 = \frac{1}{4}(x-4)$$

Q: Avg rate of change of $y=\sin x$ from $x=0$ to $x=\frac{\pi}{3}$.

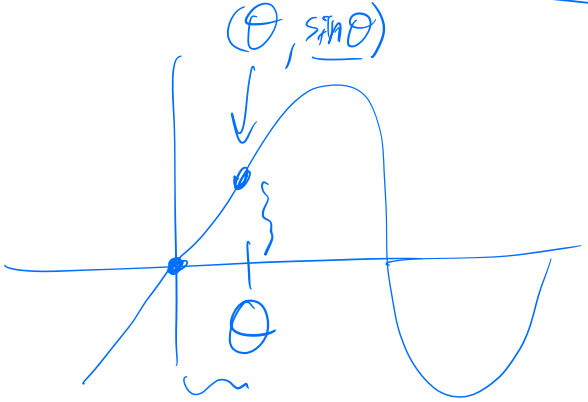


$$\text{Slope} = \frac{\frac{\sqrt{3}}{2}}{\frac{\pi}{3}}$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{3}{\pi} = \frac{3\sqrt{3}}{2\pi}.$$

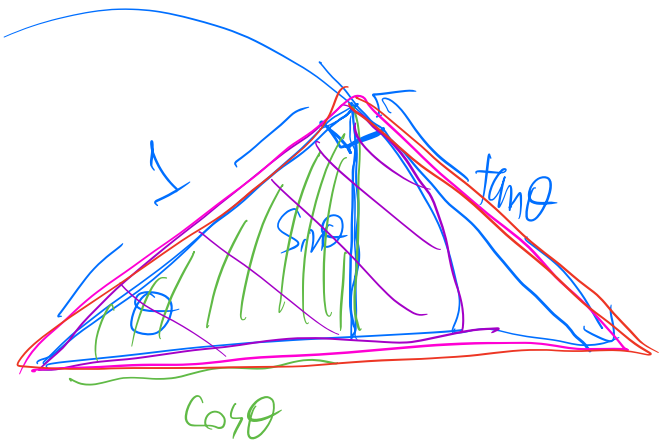


Can we figure out instantaneous rate of change of $y=\sin x$ at $x=0$.



Slope$(\theta) = \frac{\sin\theta}{\theta}$ Want:

limit as $\theta \to 0$.



Area $\cos\theta$ $\sin\theta$ $= \frac{1}{2}\sin\theta\cdot\cos\theta$

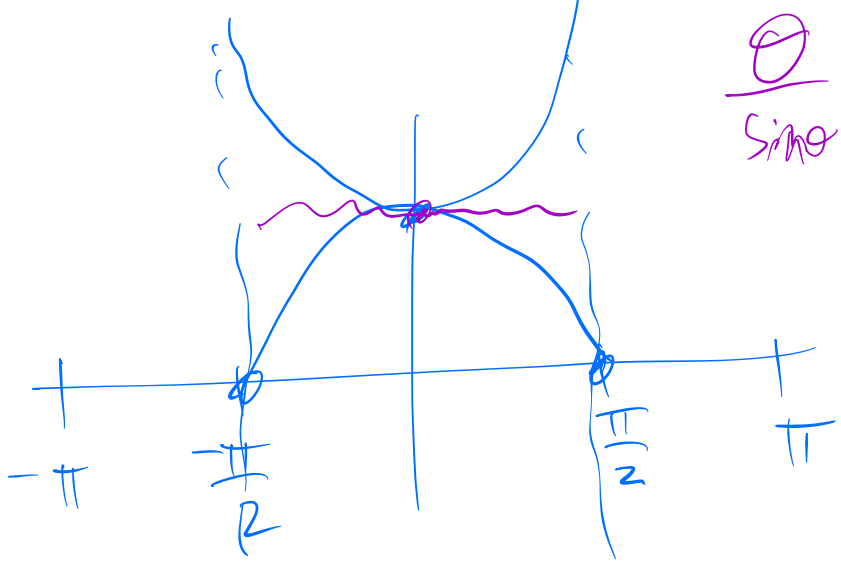
Area $= \pi\cdot 1^2\cdot\frac{\theta}{2\pi}$

$= \frac{1}{2}\theta$

Area $\tan\theta$ $= \frac{1}{2}\cdot 1\cdot\tan\theta$.

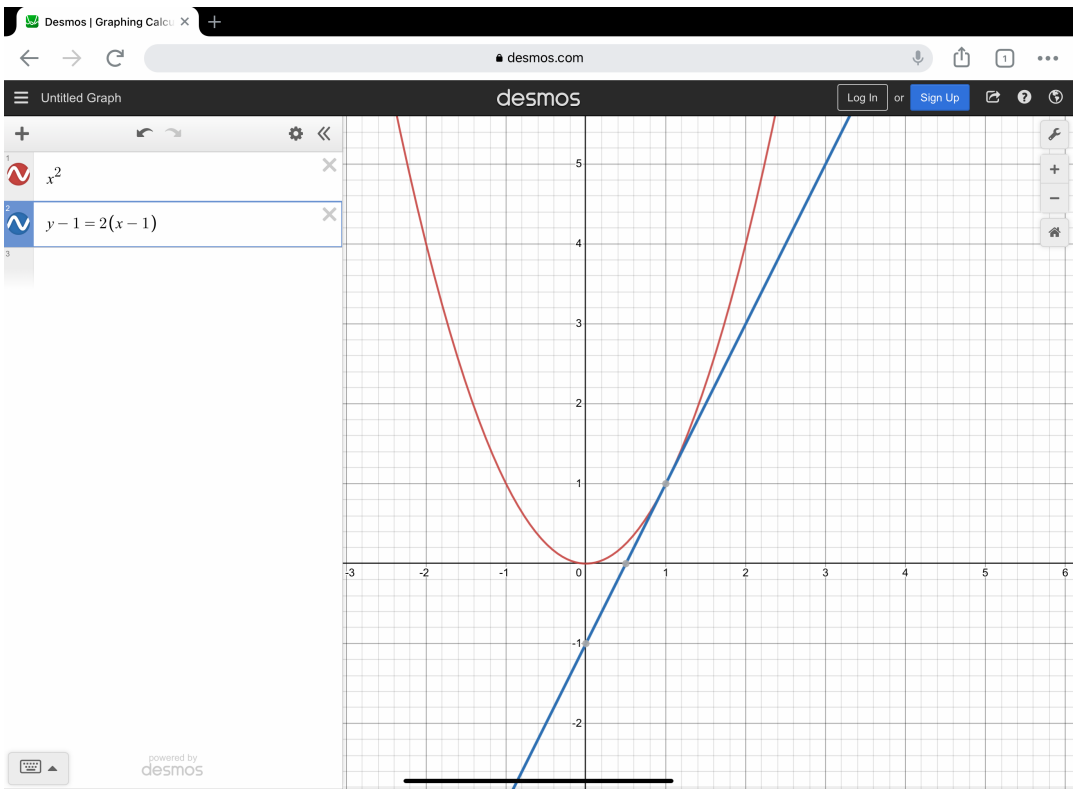
$\frac{1}{2}\sin\theta\cos\theta < \frac{1}{2}\theta < \frac{1}{2}\frac{\sin\theta}{\cos\theta}$

$\cos\theta < \frac{\theta}{\sin\theta} < \sec\theta$.



$\frac{\theta}{\sin\theta}$ is squeezed between $\cos\theta$ & $\sec\theta$.

$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$.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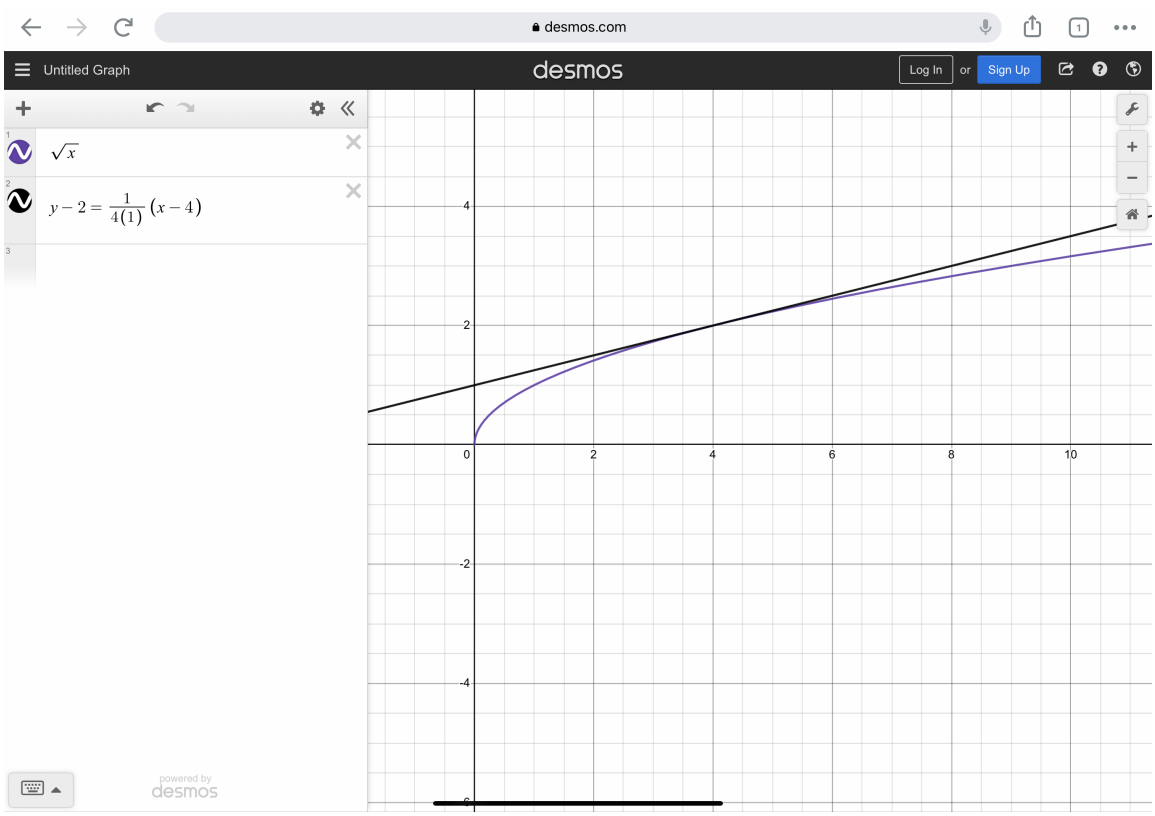
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