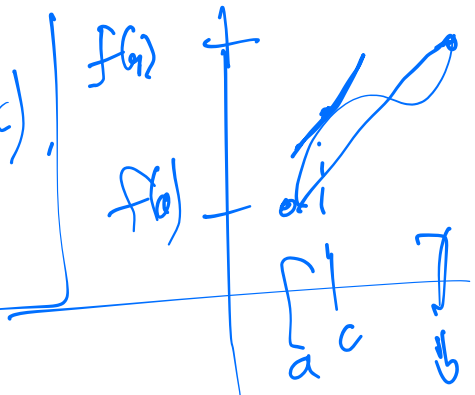
Last time: Mean Value Thm:

If f Cont on $[a,b]$ & f' exists on (a,b),

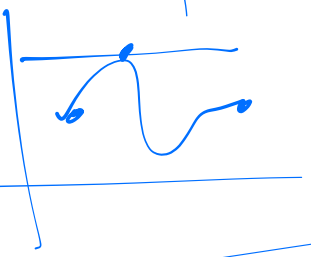
then the average rate of change is realized instantaneously somewhere, i.e.

$\exists\, c \in (a,b)$ s.t. $\frac{f(b)-f(a)}{b-a} = f'(c)$.



Rolle's Thm: $f(a)=f(b)$

Same but with $f'(c)=0$.



Ideas in Curve sketching:

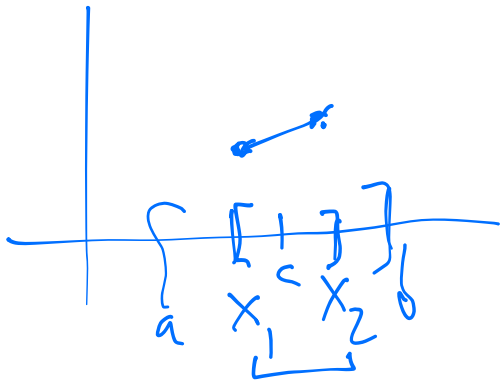
① Slope +/−, inc/dec

② Concavity, f'' +/−.

Thm: If f Cont on $[a,b]$ & f' exists on (a,b), & $\boxed{f' > 0}$, then f

is increasing on (a,b).

Pf:



Fix $a \le x_1 < x_2 \le b$

By MVT, $\exists c \in (x_1, x_2)$ s.t.

$$\frac{f(x_2)-f(x_1)}{x_2-x_1} = f'(c)$$

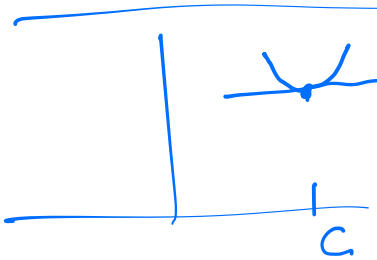
$$\Rightarrow f(x_2)-f(x_1) = \underbrace{f'(c)}_{>0}\underbrace{(x_2-x_1)}_{>0} > 0,$$

$$\Rightarrow f(x_2) > f(x_1).$$

First deriv Test: If f' changes from $+$ to $-$ at c, then c is a local max.

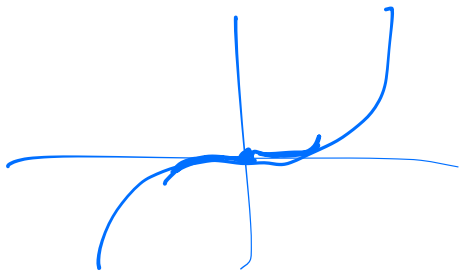
If f' changes from $-$ to $+$ ------ $-$ local min.

Second deriv test: If $f'(c)=0$, then



- $f''(c) > 0 \Rightarrow$ local min $\boxed{x^2}$ ∪
- $f''(c) < 0 \Rightarrow$ local max $\boxed{-x^2}$ ∩
- $\boxed{f''(c)=0} \Rightarrow$ no conclusion

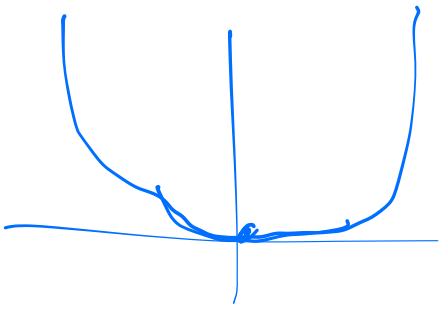
Eg: $y = x^3$



$y' = 3x^2\big|_0 = 0$

$y'' = 6x\big|_0 = 0.$

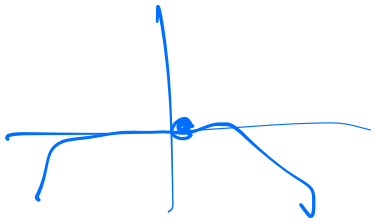
$y = x^4$



$y' = 4x^3\big|_0 = 0$

$y'' = 12x^2\big|_0 = 0.$

$y = -x^4$

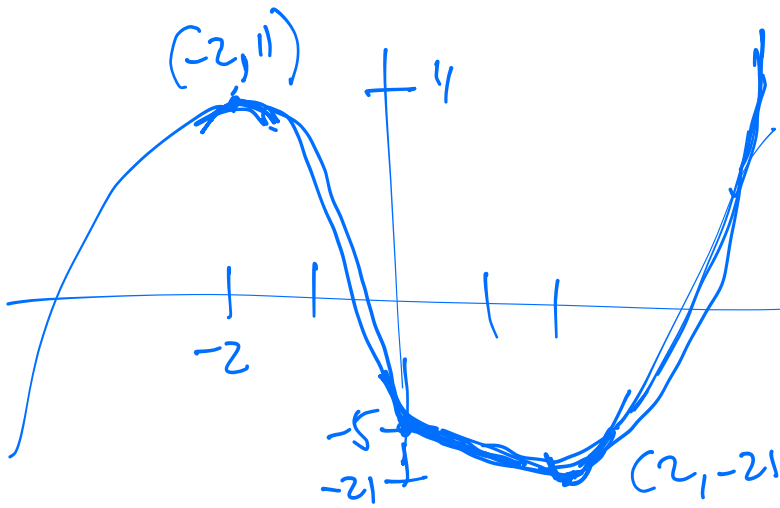


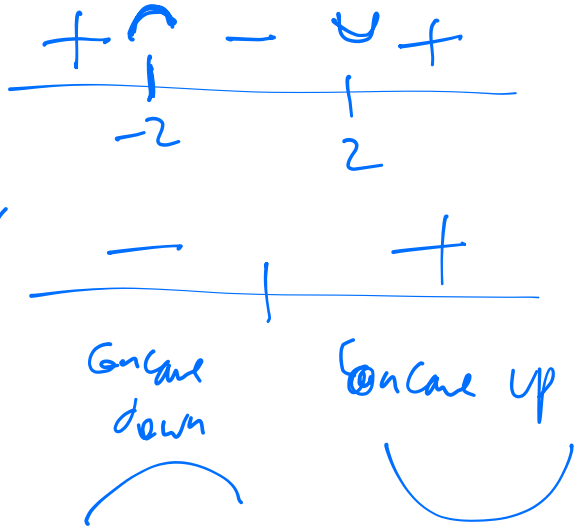
Eg.: $y = x^3 - 12x - 5$. Graph.

$y' = 3x^2 - 12 = 3(x^2 - 4) = 3(x-2)(x+2).$

$y'' = 6x.$ ← "inflection pt": $x = 0.$

Critical values: ($f' = 0$ or DNE) $c = -2, 2.$





Eg: $y = \dfrac{(x+1)^2}{1+x^2}$

$$y' = \frac{(1+x^2)(2(x+1)\cdot 1) - (x+1)^2(2x)}{(1+x^2)^2}$$

$$= \frac{(1+x)}{(1+x^2)^2}\left[2x^2 + 2 - 2x^2 - 2x\right] = \frac{2(1-x^2)}{(1+x^2)^2}.$$

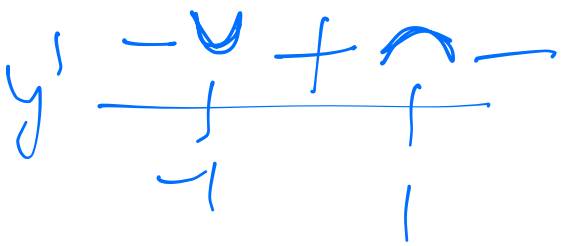
$$y'' = 2\left[\frac{(1+x^2)^2(-2x) - (1-x^2)\left[2(1+x^2)'\cdot(2x)\right]}{(1+x^2)^4}\right]$$

$$= \frac{4(1+x^2)}{(1+x^2)^{4/3}}\left[-x - x^3 - 2(x - x^3)\right]$$

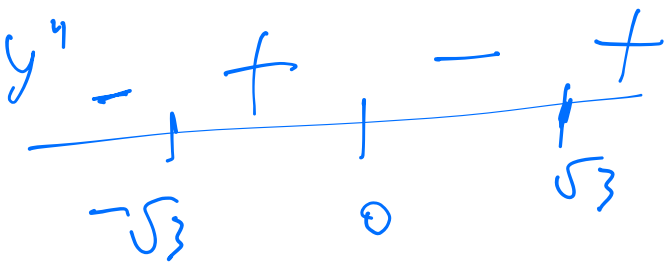
$$= \frac{4x}{(1+x^2)^3}\left[x^2 - 3\right]$$

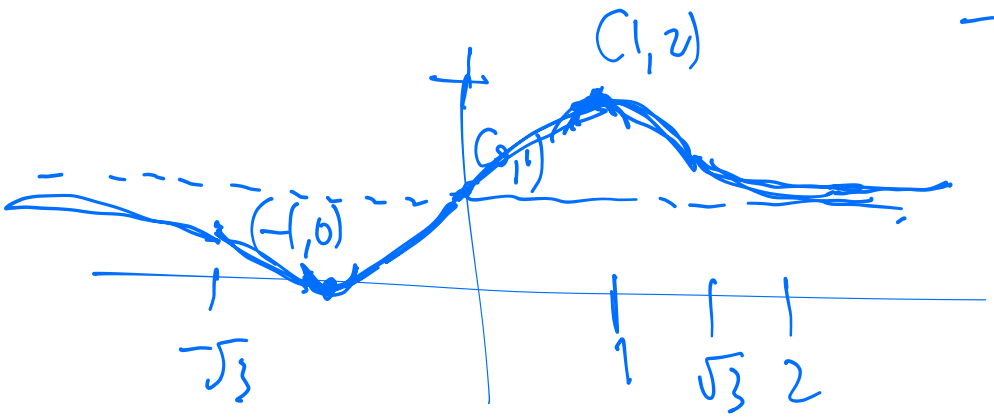
$y = \frac{(x+1)^2}{1+x^2}$, $y' = \frac{2(1-x^2)}{(1+x^2)^2}$, $y'' = \frac{4x}{(1+x^2)^3}\left[x^2-3\right]$

Crit vals: $c = \pm 1$,



inflection pts: $x = 0, \pm\sqrt{3}$





horiz asymptotes: $y = 1$

vert asympt.: None