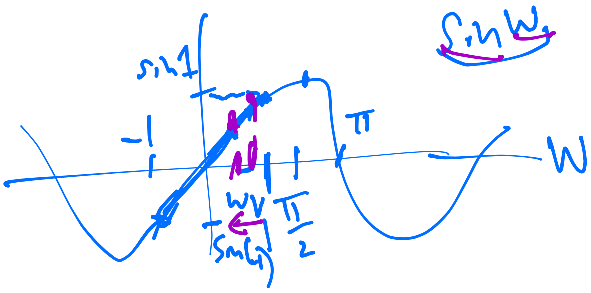
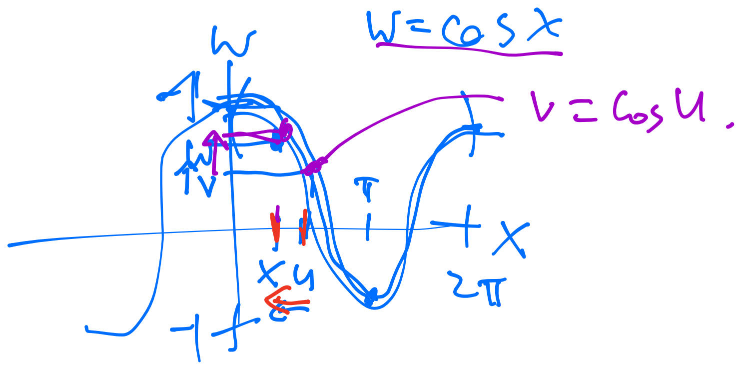
Last time: $\frac{d}{dx} e^x = e^x$, $\frac{d}{dx}\sin x = \cos x$,

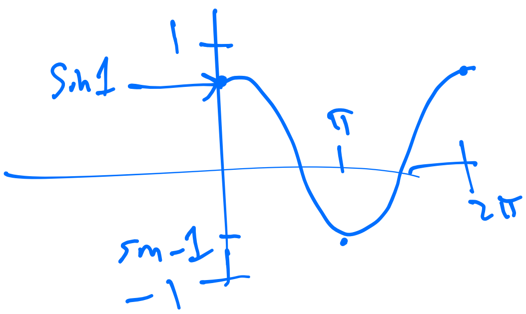
$\frac{d}{dx}(\cos x) = -\sin x$, $(f \cdot g)' = f' \cdot g + f \cdot g'$,

$\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}$, $\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$.

Eg: $f(x) = \sin(\cos x) = \sin w$







$f = \sin \circ \cos$

$f(x) = \sin(\cos(x))$. Q: $\frac{df}{dx}$?

$$\frac{df(x)}{dx} = \lim_{u \to x} \underbrace{\frac{\sin(\cos u) - \sin(\cos x)}{\cos u - \cos x}}_{\downarrow} \cdot \underbrace{\frac{\cos u - \cos x}{u - x}}_{\downarrow}$$

$W = \cos x$

Let $v = \cos u$

As $u \to x$, $v \to w$

$$\underbrace{\lim_{v \to w} \frac{\sin(v) - \sin(w)}{v - w}}_{} \to (\sin w)' \qquad \cdot \; (\cos x)'.$$

$$\frac{d}{dx}\left(\sin(\cos x)\right) = \cos(\cos x)\cdot(-\sin x).$$

"Chain Rule" $\quad [f(g(x))]' = f'(g(x))\cdot g'(x).$

$$\frac{d}{dx}\left(\underbrace{e^{(x^2+1)}}_{f(g(x))}\right) = e^{x^2+1}\cdot(2x)$$

$f(x) = e^x$

$g(x) = x^2+1$

$f'(x) = e^x$

$g'(x) = 2x$

$$\frac{d}{dx}\left(e^{2x}\right) = \frac{d}{dx}\left(e^x\cdot e^x\right) = 2e^{x}$$

$$= e^{2x}\cdot(2)$$

$[f(g(h(x)))]' = f'(g(h(x)))\cdot g'(h(x))\cdot h'(x)$

$$\frac{d}{dx}\left[\sin\left[\left(3x^2+2x+1\right)^3\right]\right]$$

$f = \sin \qquad h(x) = 3x^2+2x+1$

$g = x^3$

$$= \cos\left[\left(3x^2+2x+1\right)^3\right]\cdot 3\left(3x^2+2x+1\right)^2\cdot(6x+2).$$

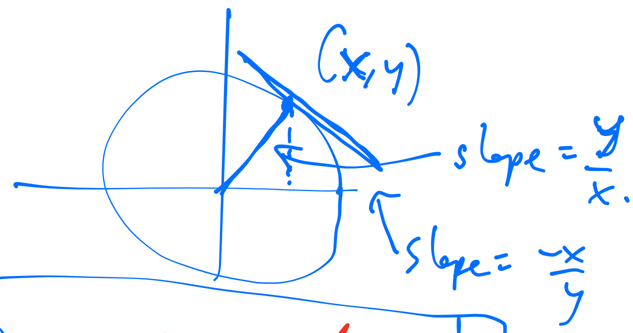
$$\frac{d}{dx}\left[(3x+1)^2\right] = 2(3x+1)'\cdot(3) = 18x+6.$$

$$\frac{d}{dx}\left[9x^2+6x+1\right] = 18x+6 \qquad [f(g(x))]' = f'(g(x))\cdot g'(x)$$

Quiz: $\frac{d}{dx}\left[\cos(2x^2+1)\right] = -\sin(2x^2+1)\cdot 4x$

Implicit Differentiation: $y = f(x)$. Secretly a function of x.

Eg: $x^2+y^2=1$

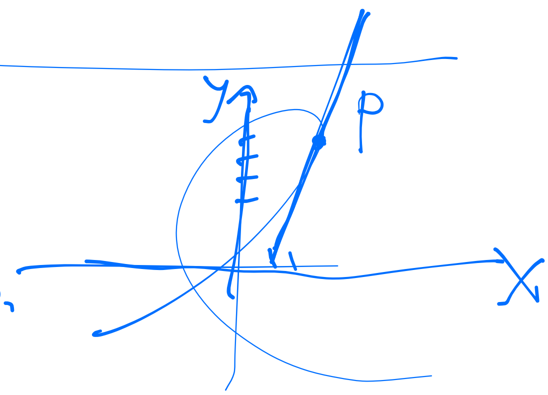


Differentiate both sides:

$$\frac{d}{dx}\left[x^2+y^2\right] = \frac{d}{dx}(1) = \boxed{0 = \cancel{2}x + \cancel{2}\cdot y\cdot \frac{dy}{dx}}$$

Solve for $\boxed{\frac{dy}{dx} = \frac{-x}{y}.}$

Eg: $x^3+y^3-9xy=0$



P: (2,4) on curve $8+64-9\cdot 8=0$.

Diff both sides.

$$0 = \frac{d}{dx}\left[x^3+y^3-9xy\right] = 3x^2+3y^2\cdot y' - 9(x\cdot y' + 1\cdot y).$$

$$9y-3x^2 = y'\left[3y^2-9x\right].$$

$$y'\Big|_{(2,4)} = \frac{9y-3x^2}{3y^2-9x}\Big|_{(2,4)} = \frac{9\cdot 4-3\cdot 2^2}{3\cdot 4^2-9\cdot 2}$$