MATH 356

Fermat's Last Theorem: 1650s (Wiles, Taylor-Wiles) '94

If $x^n + y^n \overset{(*)}{=} z^n$ & $n > 2$ $\Rightarrow$ no nontriv integer solutions. $xyz = 0$.

$(0,0,0)$

$(m,0,m)$, $(1,0,1)$, $(0,1,1)$

(if n even, $(-1,0,\pm 1)$).

(if n odd, $(-1,1,0)$).

(Diophantine Problem) ($\leftrightarrow$ polynomial equations, solved in $\mathbb{Z}$, $\mathbb{Q}$).

homogeneous. So if $\exists\,(x,y,z) \in \mathbb{Q}$ satisfying $(*)$, like $\left(\frac{7}{13}\right)^{24} + \left(\frac{9}{11}\right)^{24} = \left(\frac{5}{3}\right)^{24}$ (pretend) then multiply by denominators

$(13\cdot 3\cdot 11x,\ 13\cdot 3\cdot 11y,\ 13\cdot 3\cdot 11z) \in \mathbb{Z}$.

FLT $(n=4)$ $x^4 + y^4 = z^4 \Rightarrow xyz = 0$.

What about $x^2+y^2=z^2$ (Pythagorean thm.)

Thm (ancient): If $x^2+y^2=z^2$ & primitive [i.e. $\nexists\, d \mid x$ & $d \mid y$ & $d \mid z$] then z = odd, assume x = odd & y = even. & $\exists\, r, s \in \mathbb{Z}$, coprime, one even, one odd. So that

$$x = r^2 - s^2,\ y = 2rs,\ z = r^2 + s^2.$$

$(10, 24, 26)$. $10^2 = 100$, $24^2 = 576$, $26^2 = 676$. Not primitive.

odd

$\to (5, 12, 13)$. $25 + 144 = 169$, yes primitive.

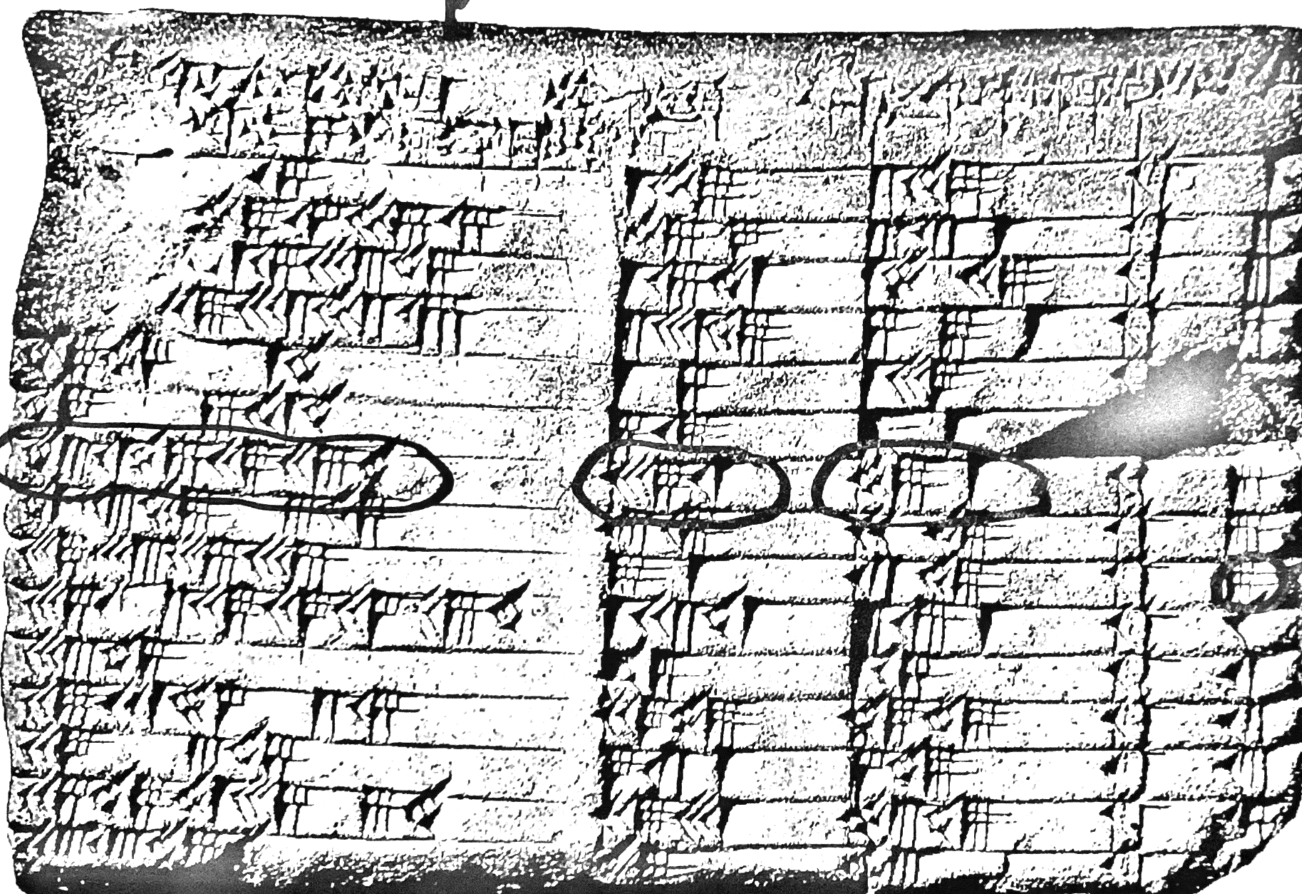
Try $r = 3, s = 2$; $\to$ $3^2 - 2^2 = 5 = x$ ✓.

$2 \cdot 3 \cdot 2 = 12 = y$ ✓.

$3^2 + 2^2 = 13 = z$ ✓.

Exercise 1 If $x = r^2 - s^2$, $y = 2rs$, $z = r^2 + s^2 \Rightarrow x^2 + y^2 = z^2$.

Plimpton 322. Babylonian Tablet ~1800 BCE.



7.

9.

10.

Line 7, column 3:

$59 \cdot 60 + 1 = \underline{3541} = 59, 1$

Line 7, column 2: <<< , <1

$38 \cdot 60 + 11 = 2291 = 38, 11$

$3541^2 = 12,538,681$ — d^2

$-2291^2 = -5,248,681$ — w^2

$2700^2 = 7,290,000$ — ℓ^2

$\boxed{\text{Exercise 2:}}$ Work out line 8.

By the way, column 1: d^2/ℓ^2.

1, 43, 11, 50, . . .

$$\frac{12538681}{7290000}$$

$$1 + \frac{43}{60} + \frac{11}{60^2} + \frac{50}{60^3} + \ldots = 1.71998$$