Oral Exam Syllabus

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1 Combinatorics

Basic Enumeration: counting arguments, generating functions, recurrence relations, reflection principle, inclusion-exclusion

Permutations & Tableau: Robinson-Schensted Correspondence, Knuth relations, Greene's Theorem, the Hook Length Formula, the Marcus-Tardos Theorem (Stanley-Wilf Conjecture), generating trees, WQO for permutations

Hypergeometric functions & identities: definition and representation, notion of q-analog and examples, Sister Celine's Algorithm, Gosper's Algorithm, Zeilberger's Algorithm

Extremal Set Theory: Ray-Chaudhuri Wilson and variants, Sperner's theorem, Erdős-Ko-Rado, Borsuk conjecture and Kahn-Kalai counterexample, Harper's Theorem, Kruskal-Katona

Lattices & Posets: Dilworth's Theorem, Fundamental Theorem of Finite Distributive Lattices, Birkhoff covering property, Jordan-Dedekind chain condition, Möbius inversion, Weisner's Theorem, Dowling-Wilson Theorem

Correlation Inequalities: Harris-Kleitman, FKG, four functions, application to xyz inequality, BK inequality, stochastic domination

Ramsey Theory: Ramsey's Theorem, infinite Ramsey theory, probabilistic lower bounds, stepping-up lemma, van der Waerden, $R(3, n)$ upper and lower bounds

Discrepancy: Erdős-Selfridge Theorem, Beck-Fiala Theorem, discrepancy in arithmetic progressions, linear and hereditary discrepancy, 6 standard deviations suffice, Komlós Conjecture

Algebraic Methods: Combinatorial Nullstellensatz, Chevalley-Warning Theorem, Cauchy-Davenport Theorem, Frankl-Wilson Theorem, bounds on the Shannon capacity using dimension arguments

2 Graph Theory

Matching: Hall's Theorem, bipartite matching algorithm, König's Theorem, Tutte's 1-factor Theorem, matching polytope

Connectivity: Menger's Theorem, Max-Flow-Min-Cut Theorem, Kruskal's Algorithm

Planarity: Euler's formula, Kuratowski's Theorem, Wagner's Theorem

Coloring: Brooks' Theorem, Vizing's Theorem, 5 Color Theorem, perfect graphs, Weak Perfect Graph Theorem

Extremal Problems: Turán's Theorem, statement of Regularity lemma and application to Erdős-Stone Theorem

3 Probabilistic Methods

Basics: Stirling's formula, Bonferroni inequalities, Chebyshev's inequality, Chernoff bound

Alterations: application to lower bound on property B

Second Moment Method: general procedure, application to threshold function for having a certain graph as a subgraph, 2^{nd} moment method for nonnegative random variables

Lovász Local Lemma: symmetric and general versions, applications to linear arboricity conjecture and Latin transversals

Martingales: Azuma's inequality, edge and vertex exposure, applications to chromatic number

Poisson Paradigm: Janson inequalities, application to number of triangles in $G_{n,p}$, Brun's sieve, application to number of isolated points

Random graphs: monotone properties, $G_{n,p}$ versus $G_{n,M}$, existence of threshold functions, relationship between connectedness and having no isolated vertices, probabilistic refutation of Hajós's Conjecture

4 Probability

Probability Spaces
Random Variables
Borel-Cantelli Lemma
Laws of large numbers
Law of iterated logarithm
Central Limit Theorem
Conditional Expectation
Random Walks & Polya's Theorem
Arc-Sine Laws
Kolmogorov's Zero-One Law
Are you paying attention?
Percolation Theory:
Bond percolation
Continuity properties of $\theta(p)$
Uniqueness of infinite cluster in L^d.