Oral Qualifying Examination Syllabus

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March 9, 2006

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Experimental mathematics

Basic enumeration: counting arguments; inclusion-exclusion principle; generating functions; Bell numbers; Catalan numbers; Eulerian numbers; Fibonacci numbers; Stirling numbers

Combinatorics of subwords: the number of words of length l on k letters that do not contain a given subword; the number of occurrences of a given subword in the set of words of length l on k letters

Geometry of numbers: sums of two and four squares; Cauchy's polygonal number theorem

Calculus of finite differences: finite difference operator; indefinite summation; Bernoulli polynomials; Gosper's algorithm; (homogeneous and inhomogeneous) linear difference equations with constant coefficients; Petkovšek's algorithm

Hypergeometric identities: fundamental theorem; Fasenmyer's algorithm; Zeilberger's algorithm; Wilf–Zeilberger pairs; companion identity and dual identities

Digit sequences: binary exponentiation; Gray code; Johnson–Trotter algorithm; Thue–Morse sequence

Binomial coefficients modulo p^q: Kummer's theorem; Lucas' theorem; Granville's theorem; fractal dimension of Pascal's triangle modulo p

One-dimensional cellular automata: Jen's aperiodicity theorem; generalized linearity; positional bijectivity and reversibility; global and local nestedness

Diophantine approximation

Approximability by rationals: continued fractions; Dirichlet's theorem; best approximations; Hurwitz's theorem; Liouville's theorem; quadratic irrationals; Pell equation

Markoff spectrum: Markoff equation; gaps; Hall's ray

Transcendence theory: irrationality of e, $e^{m/n}$, π, and $\zeta(3)$; transcendence of e and π; Lindemann–Weierstrass theorem; statement of Gelfond–Schneider theorem

Distribution modulo 1: Weyl criterion; uniform distribution of $\{n\alpha\}$ for irrational α; uniform distribution of polynomials; characterization of normal numbers

Integer relation algorithms: LLL algorithm; PSLQ algorithm