Oral Qualifying Exam Syllabus

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Committee (in alphabetical order): R. Bumby, H. Iwaniec, J. Tunnell.

1. Analytic Number Theory
   - (a) Analytic properties of L-functions and the Riemann zeta functions
   - (b) Primes in arithmetic progression
   - (c) Siegel zero problem
   - (d) Prime number theorem and prime number theorem for arithmetic progressions

2. Algebraic Number Theory
   - (a) Invariants of number fields: rings of integers, discriminants and orders
   - (b) Arithmetic of number fields: splitting of primes, ramification
   - (c) Class groups
   - (d) Structure of units in number rings

3. Elliptic Curves
   - (a) Elliptic curves over the complex field: elliptic functions and the j-function
   - (b) Elliptic curves over finite fields
   - (c) Hasse-Weil L-functions of elliptic curves

4. Modular Forms
   - (a) Modular Forms for the full modular group and its congruence subgroups
   - (b) Eisenstein series
   - (c) Structure of the ring of modular forms
   - (d) Hecke operators

5. Elliptic Functions
   - (a) The elliptic functions
   - (b) The Weierstrass Function, its differential equation, and a parameterization of the cubic.
   - (c) The elliptic integrals
   - (d) Addition theorems for the elliptic integrals $F(\Phi)$ and $E(\Phi)$
   - (e) The elliptic Jacobi functions
   - (f) The Weierstrass theorem on functions possessing an algebraic addition theorem