ORAL EXAM

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Experimental Mathematics, Combinatorics and Graph theory

1. **Experimental Mathematics and Combinatorics**

   Hypergeometric Functions: definition and representation, notion of q-analog and examples.

   The five basic algorithms for hypergeometric identities: Mary Celine Fasenmyer's algorithm, Gosper's algorithm, Zeilberger's algorithm, Petkovsék's algorithm, the WZ phenomena, Dual(Shadow) and the Companion identity and the Fundamental Theorem for hypergeometric identities.

   The (Umbral) Transfer-Matrix-Method and applications to enumeration: Finite and infinite case.

   Integer relation algorithms and applications: The LLL based Algorithm.

   Basic enumeration: Counting arguments, Generating functions, The Lagrange inversion formula, P-recursive sequences, Stirling numbers, Bell numbers, Catalan numbers and Inclusion-Exclusion Principle.

2. **Graph Theory**

   The basics of Graph theory: graph parameters, trees, bipartite graphs, Eulerian tour, paths and cycles.

   Matching: König's theorem, Hall's theorem, Tutte's theorem, The Gallai and Milgram theorem, Path covers and Dilworth's theorem.

   Connectivity: the structure of 2-connected and 3-connected graphs, k-connected graphs, Menger's theorem, max-flow-min-cut theorem, statement of Mader's theorem, Edge-disjoint spanning trees and paths between given pair of points.

   Planar graphs: Euler's formula, Kuratowski's theorem, plane duality, and abstract duality.

Coloring: Vertex coloring, Edge coloring, Five color theorem, Brook's theorem, König's theorem, Vizing's theorem, Weak perfect graph theorem, Strong perfect graph theorem and Lovăsz's theorem.

Probabilistic Method (Random graphs): Stirling's formula, G(n,p) verses G(n,M), monotone properties, properties of almost all graphs, Erdös and Reyni's theorem, threshold function for having a certain graph as a subgraph, relationship between being connected and having no isolated vertices.

Hamiltonian Cycles: Dirac's theorem, Ore's theorem, Hamiltonian and degree sequences(Chvátal's theorem).

Extremal problems: Turán's theorem, statement of regularity lemma and how it is applied.

Minors, Trees, and WQO: Well-quasi ordering, the graph minor theorem for trees, the graph minor theorem and implications for embedding graphs on surfaces.

Matroids: Definitions of matroids, The Matroid Intersection theorem, The Matroid Sum theorem, The packing theorem, The covering theorem and applications to graph theory.

3. **Probability theory**
Probability space, random-variable, linearity of expectation, conditional probability, Chebyshev's inequality, Markov's inequality, the Lovász local lemma, Central limit theorem, Law of large numbers.

4. **Ramsey theory**
Definition and existence of Ramsey numbers, Original Ramsey Theorem, Infinite Ramsey theorem, Ramsey numbers: $R(3,3)$,$R(3,4)$,$R(4,4)$,$R(2,k)$ Chvátal-Rödl-Szemerédi-Trotter theorem, Bounds on the Ramsey numbers, ven de Waerden theorem, Schurs triples, Röth's theorem for arithematic progression(statement) and the statement of Szemerédi's theorem for arithmetic progression of arbitrary length.