Oral Exam Syllabus

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1 Functional Analysis

1.1 Topological Vector Spaces

A. Basic Properties and Examples
B. Locally Convex Spaces
C. Banach Spaces

1.2 Baire Category Theorem and Consequences

A. Baire Category
B. Uniform Boundedness Principle
C. Open Mapping Theorem
D. Closed Graph Theorem

1.3 Duality and Convexity

A. Hahn-Banach Theorems
B. Banach-Alaoglu Theorem
C. Krein-Milman Theorem

1.4 Fixed Point Theorems and Applications

A. Basic Fixed Point Theorems (Brouwer and Contraction Principle)
B. Markov-Kakutani
C. Local Solutions of ODE

1.5 Hilbert Spaces

A. Geometry of Hilbert Space
B. The Algebra of Operators
C. Spectral Theorem for Hermitian and Normal Operators
D. Unbounded Operators

2 Harmonic Analysis

2.1 Fourier Transform

A. Construction and Basic Properties
B. Application to L^2
C. Schwartz Space
D. Tempered Distributions
E. Operators that Commute with Translations

2.2 Some Fundamental Notions

A. The Hardy-Littlewood Maximal Function
B. Lebesque Differentiation Theorem
C. Vitali Covering Lemma
D. Marcinkiewicz Integral
E. Calderon-Zygmund Decomposition

2.3 Interpolation of Operators

A. Riesz-Thorin
B. Marcinkiewicz Interpolation Theorem

2.4 Singular Integrals

A. L^P Boundedness
B. Hilbert Transform
C. Riesz Transform
D. Applications

Primary References:
Rudin, *Functional Analysis*
Stein, *Singular Integrals and Differentiability Properties of Functions*
Stein and Weiss, *Introduction to Fourier Analysis on Euclidean Spaces*