Oral Exam Syllabus

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General/Combinatorial Group Theory

Free Groups and Their Subgroups

- Definition and elementary properties of free groups
- Nielsen-Schreier Theorem, Takahasi Theorem, Levi Theorem
- Properties of subgroups of finite index

Ref: Magnus 1.4, 2.4; Lyndon 1.1, 1.3

Automorphisms of Free Groups

- Presentation and properties of $Aut(F)$

Ref: Lyndon 1.4; Magnus 3.5

Nielsen Transformation

- Definition and applications
- Generalized word problem for (finitely generated) H in G

Ref: Magnus 3.1, 3.2; Lyndon 1.2

Tietze Transformation

- Definition and applications

Ref: Magnus 1.5

Free Products, Free Products with Amalgamation, HNN Extensions

- Definitions and properties
- The normal forms for free products, free products with amalgamation and HNN extensions
- The conjugacy theorems for free products, free products with amalgamation, and HNN extensions
- The Higman, Neumann, and Neumann theorem

Ref: Magnus 4.1, 4.2; Lyndon 1.11, 4.2

Equations over Groups, Equations in Free Groups

Ref: Lyndon 1.6, 1.8

Computational Group Theory

Rewriting Systems

- Orderings of free monoids, Canonical words, A test for confluence
- The Knuth-Bendix procedure
- Heuristics, Right Congruences

Ref: Sims Ch. 2

Subgroups of Free Products of Cyclic Groups

- Definition of languages and automata, Types of automata
- Niladic rewriting system, Subgroups and their languages, Important cosets, Coset automata
- Basic coset enumeration
- The coincidence procedure, Standardization, Computation with subgroups, Standard coset tables

Ref: Sims Ch. 3, Ch. 4

Coset Enumeration

- The general case
- The HLT strategy, The Felsch strategy, Standardizing strategies, Ten versions
- Low-index subgroups
- A comparison with the Knuth-Bendix procedure

Ref: Sims Ch. 5

The Reidemeister-Schreier Method

- Presentations of subgroups
- Examples of extended coset enumeration, An extended HLT enumeration procedure
- Symplifying presentations

Ref: Sims Ch. 6, Magnus 2.3

Abelian Groups

- Free abelian groups, Elementary matrices, Finitely generated abelian groups
- Modular techniques, The Kannan-Bachem algorithm
- Lattice reduction, The modified LLL algorithm, Comparison

Ref: Sims Ch. 8