Oral Qualifying Exam Syllabus

Amit Priyadarshi
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Committee: Roger Nussbaum, Richard Bumby, Michael Vogelius, Richard Wheeden

I. **Major Topic: Fractal Sets and Dimensions**

1. **Introduction**
   - Definition of Hausdorff measure and Hausdorff dimension
   - Basic properties
   - Vitali covering theorem
   - Relationship between n-dimensional Hausdorff measure and Lebesgue measure
   - Calculation of Hausdorff dimension and measure for Cantor set
2. **Self-similar sets and Hutchinson's theorem**
   - Contractions and Similitudes
   - Invariant sets and self-similar sets
   - Open set condition
   - The Hutchinson's theorem
3. **Apollonian packings**
   - Lower and upper bounds for the Hausdorff dimension of the residual set for the Apollonian packing
   - The disk-packing constant
4. **Basic density properties**
   - Upper and lower densities
   - Regular and irregular sets
   - Elementary density bounds

II. **Minor Topic: Linear Operator Theory**

1. **Bounded Linear operators on Hilbert spaces**
   - Some examples of bounded linear operators with estimates of norms

- Bounded linear functionals and the Riesz representation theorem
- Self adjoint and normal operators
- Compact operators

2. **Spectral Theory**
   - Spectrum and resolvent
   - Spectral radius
   - Functional Calculus
   - Spectral theorem for compact self adjoint operators
   - Spectral theorem for compact normal operators
   - Spectral theorem for bounded self adjoint operators
3. **Trace class and Hilbert-Schmidt operators**
4. **Trace and determinant**

References

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[B] Boyd, D.W., *The Disk-Packing Constant*, Aequationes Mathematicae 7, 182-193, 1971.

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[GG] Gohberg, I., Goldberg, S., *Basic Operator Theory*, Birkhäuser, 1981.

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