Oral Exam Syllabus
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April 2007

I. Functional Analysis
- A. Hilbert Spaces
  - i. Riesz Representation
  - ii. Orthonormal Bases
  - iii. Spectral Theory for Compact and Self-Adjoint operators
- B. Banach Spaces
  - i. Baire-Category
  - ii. Open Mapping, Closed Graph, Uniform Boundedness
- C. Weak Topologies, Weak* Topologies
  - i. Banach-Alaoglu Theorem

II. Partial Differential Equations
- A. Standard Equations (Laplace, Wave, Heat)
  - i. Maximum Principles, Mean Value
  - ii. Harnack's Inequality
  - iii. Gradient Estimates
- B. Sobolev Spaces
  - i. Definitions, H^s, $W^{k,p}$
  - ii. Density of smooth functions in $W^{k,p}$
  - iii. Sobolev Inequalities
  - iv. Trace Theorems/Embedding Theorems
- C. General 2nd Order Elliptical Equations
  - i. Maximum principle, Hopf Lemma
  - ii. Weak solutions (definition, existence, Fredholm Alternative)
  - iii. Solution regularity, Schauder estimates (L^p based spaces)

III. Numerical Analysis
- A. Finite Difference Methods
  - i. Laplace's Equation (different boundary conditions)
  - ii. Discrete Maximum Principle
  - iii. Error Analysis
- B. Finite Element Methods
  - i. Variational Formulations
  - ii. Error estimates for Laplace's Equation (different boundary conditions)

References

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Evans, Partial Differential Equations, AMS, Rhode Island, 2002.

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Folland, Real Analysis, Wiley, New York, 1999.

Reed and Simon, Functional Analysis, Elsevier, Singapore.

Rynne and Youngson, Linear Functional Analysis, Springer, Great Britain, 2001.

Wheeden and Zygmund, Measure and Integral, Marcel Decker, New York, 1977.