Math 16:642:611

Topics in Applied Mathematics - Obstacle problems – Fall 2014

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Summary: The goal of the course is to introduce graduate students to methods for solving variational inequalities, obstacle problems, and free boundary problems and their applications.

Applications of obstacle problems arise in many areas of pure and applied mathematics and engineering, including mathematical finance – in particular, the American-style option pricing problem. Examples drawn from other areas of pure and applied mathematics based on the interests and background of students. Our emphasis will be on the universal applicability of the methods introduced in the course to a wide variety of examples in all areas of pure and applied mathematics.

Topics selected will depend on the interests and backgrounds of the audience, but may include

- Obstacle problems, variational inequalities, and free boundary problems
- The American-style option pricing problem and other examples from applied mathematics
- Existence, uniqueness, and regularity of solutions to variational inequalities and obstacle problems
- Optimal regularity of solutions to obstacle problems near the free boundary
- Numerical solution of obstacle problems by finite difference and finite element methods
- Introduction to viscosity solutions for non-linear partial differential equations

Students will be polled at the start of the class regarding their interests.

Audience:

- Doctoral students in pure or applied mathematics as well as doctoral students in engineering who are interested in partial differential equations and their applications;
- Master's students in mathematical finance with a good undergraduate mathematics background.

The choice of topics and the level of coverage will be carefully adjusted for the audience at the beginning of the semester in order to accommodate different interests and backgrounds.

Prerequisites:

An undergraduate (or higher-level) course on real analysis covering elementary measure theory (Lebesgue integral), and the concepts of Hilbert spaces and Banach spaces will be useful.

Co-requisites:

A one-semester undergraduate course on partial differential equations (for example, based on the text by Walter Strauss) or a graduate level course on partial differential (for example, based on the text by Lawrence Evans) would be useful, but will not necessarily be assumed.

Grading:

This is an elective course for doctoral students and master's degree students. There are no formal course requirements (homework or exams) and the only requirement for a perfect grade is attendance.

I will occasionally provide optional homework problems (which may be theoretical or involve use of open source packages for solving partial differential equations and obstacle problems), but they will not be required.

Textbooks:

1. P. Feehan, *Lectures on variational inequalities, obstacle, and free boundary problems in mathematical finance*, Rutgers University, Fall 2011 and 2013, Columbia University, Spring 2013.
2. J-F. Rodrigues, *Obstacle problems in mathematical physics*, North-Holland, New York, 1987.

Primary references:

1. A. Friedman, *Variational principles and free boundary problems*, Dover, New York, 2010.
2. R. Glowinski, J-L. Lions, and R. Trémolières, *Numerical analysis of variational inequalities*, North-Holland, 1981.
3. D. Kinderlehrer and G. Stampacchia, *An introduction to variational inequalities and their applications*, Academic, New York, 1980.
4. S. Koike, *A beginner's guide to the theory of viscosity solutions*, MSJ Memoirs, vol. 13, Mathematical Society of Japan, Tokyo, 2004; second edition, 2010, available at rimath.saitama-u.ac.jp/lab.jp/skoike/book/evisbyMizuno.pdf.
5. A. Pascucci, *PDE and martingale methods in option pricing*, Springer, New York, 2011.
6. N. Hilber and O. Reichmann and C. Schwab and C. Winter, *Numerical solution of partial differential equations in financial engineering: finite element and finite difference methods*, Springer, New York, 2013.
7. G. M. Troianiello, *Elliptic differential equations and obstacle problems*, Plenum, New York, 1987.

Supplementary references:

1. M. G. Crandall, *Viscosity solutions: a primer*, in “Viscosity solutions and applications” Lecture Notes in Math., vol. 1660, Springer, 1997, pp. 1-43.

2. M. G. Crandall, H. Ishii, and P-L. Lions, *User's guide to viscosity solutions of second order partial differential equations*, Bull. Amer. Math. Soc. **27** (1992), 1-67.

3. L. C. Evans, *Partial differential equations*, second edition, American Mathematical Society, Providence, RI, 2010.

4. A. Friedman, *Partial differential equations*, Dover, New York, 2008.

5. A. Friedman, *Partial differential equations of parabolic type*, Dover, New York, 2008.

6. D. Gilbarg and N. S. Trudinger, *Elliptic partial differential equations of second order*, Springer, New York, 2001.

7. R. Glowinski, *Lectures on numerical methods for non-linear variational problems*, Springer, 2008.

8. Q. Han and F. Lin, *Elliptic partial differential equations*, Courant Lecture Notes, American Mathematical Society, Providence, RI, 2011.

9. N. V. Krylov, *Lectures on elliptic and parabolic equations in Hölder spaces*, American Mathematical Society, Providence, RI, 1996.

10. N. V. Krylov, *Lectures on elliptic and parabolic equations in Sobolev spaces*, American Mathematical Society, Providence, RI, 2008.