Syllabus for 640:519 (Spring 2013)
Nonlinear Partial Differential Equations and Hamiltonian Systems; Existence Theorems
Instructor: A. Bahri

Aim of the course: this course should be a preparation to more advanced courses in Nonlinear Analysis, Differential Geometry and Dynamical Systems.

Prerequisite: This course is a continuation of 509. The assumption is that the students know the basic facts about Sobolev spaces and injections, degree theory and critical point theory. For these two latter subjects, adjustments can be made for new students. Knowledge of the basic facts about Sobolev spaces is a prerequisite.

The aim of this course is to apply these techniques (degree theory, critical point theory) to Nonlinear Partial Differential Equations and to Hamiltonian Systems. The outline of the course is as follows:

1. Asymptotically linear Partial Differential Equations: $-\Delta u = g(x,u), u_{|\partial\Omega} = 0$. Ambrosetti-Prodi type results will be proven.

2. Bounded or Asymptotically linear Hamiltonian systems: $\dot{z} = JH'(t,z), z(0) = z(T)$. The framework of the Conley-Zehnder theorem will be introduced. The final existence argument (using Conley index) for the proof of the Arnold conjecture on tori will be outlined.

3. Superlinear and subcritical Nonlinear Partial Differential Equations: $-\Delta u = g(x,u), u_{|\partial\Omega} = 0; \Omega \subset \mathbb{R}^n$, with $\lim_{s \longrightarrow \pm\infty} \frac{g(x,s)}{s} = \infty; |g(x,s)| \leq C|s|^q + C$, and $q < \frac{n+2}{n-2}$. The perturbation theory (with applications) from the odd case $g(x,u) = -g(x,-u)$ will be studied in particular.

4. Superlinear Hamiltonian Systems:

(i) P.H.Rabinowitz periodic orbit theorem

(ii) Existence of infinitely many solutions to $\dot{z} = JH'(z), z(0) = z(T); \lim_{|z| \longrightarrow \infty} \frac{H(z)}{|z|^2} = \infty$

(iii) Perturbation Theory for $\dot{z} = JH'(z) + f(t), z(0) = z(T)$

5. The Palais-Smale condition for the Yamabe problem and for Yamabe-type problems: $-\Delta u + q(x)u = K(x)u^{\frac{n+2}{n-2}}, x \in M^n$. This will be covered if time permits.