Contents of Math 501

501 deals with both the general theory of measure and integration and with the special cases of Lebesgue measure and Borel measures on $\mathbb{R}^n$ per se. The particular emphasis (general case versus the special cases) may vary depending on the professor. In general, students will get some good idea of the general picture and will work extensively with Lebesgue measures and integrals.

The following topics are core topics.

- σ-algebra of sets; Borel sets; measurable functions and their closure properties under various operations and limits; measurable functions.
- Countably additive measures; measure spaces; Caratheodory's extension theorem.
- Construction of Lebesgue measure; Lebesgue measurable sets, existence and construction of non-Lebesgue measurable sets.
- Definition of the Lebesgue integral (or more generally the integral with respect to a measure) and its basic theory, especially for convergence of integrals: Fatou's lemma, the Monotone Convergence Theorem, and the Dominated Convergence Theorem.
- Convergence of sequences of functions: convergence in measure, convergence almost everywhere, convergence in L^p norm, and their interrelationships; completeness of $L^p(\mathbb{R})$, Egorov's theorem, Lusin's theorem.
- Product measures and the Fubini-Tonelli theorem.

Other important topics that definitely appear in 501 or 502 are

- Signed measures and the Radon-Nikodym theorem. Singular measures.
- Functions of bounded variation and absolutely continuous functions on $\mathbb{R}$.
- Differentiation of measures, maximal inequality.

The distribution of these topics between 501 and 502 varies somewhat from year-to-year, and the 501 instructor may add some additional topics. Students who request and receive an exemption from 501 and take 502 should make sure they know the optional topics that were covered in 501 so they can review them on their own if necessary.

The major goals of 501 is for the student to have a thorough understanding of the basic definitions, the principal examples, the main theorems (as well as their proofs) and the main techniques, and to be able to apply them fluently.