Mathematics 551 **Algebra** **Fall 2006**

Week 9 Modules
Sections 4.1-4.3

1. Let M be a left R-module and let $B = \{b \in R | bx = 0 \text{ for all } x \in M\}$ be the annihilator of M. Show that B is an ideal of R. Show that if C is any ideal contained in B then M is a left R/C module under $(a + C)x = ax$.

2. (Schur's Lemma) Recall that a module M is irreducible (or simple) if the only submodules are 0 and M.
   a) Show that every irreducible module is cyclic
   b) Show that any nonzero homomorphism from an irreducible module M_1 to an irreducible module M_2 is an isomorphism. Conclude that if M is irreducible, the endomorphism ring $End_R(M)$ is a division ring.

3. Let **Top** be the category of all topological spaces, with morphisms the continuous maps. Let F be the forgetful functor to the category of sets. Determine the left adjoint and the right adjoint of F.

4. Let $\mathcal{J}$ be an index category, and let $\mathcal{C}$ be another category. Let $\mathbf{FUNCT}(\mathcal{J}, \mathcal{C})$ be the category of functors from $\mathcal{J}$ to $\mathcal{C}$. Consider the functor Δ from $\mathcal{C}$ to $\mathbf{FUNCT}(\mathcal{J}, \mathcal{C})$ which assigns an object C of $\mathcal{C}$ to the constant functor in $\mathbf{FUNCT}(\mathcal{J}, \mathcal{C})$ with value C. Show that the left adjoint of Δ (if it exists) is the functor assigning to each functor F from $\mathcal{J}$ to $\mathcal{C}$ the inductive limit of F. Similarly, identify the right adjoint of Δ.

5. Hungerford 4.1.7

6. Hungerford 4.2.13

7. Hungerford 4.3.1

8. Hungerford 4.3.3

9. Hungerford 4.3.5