Mathematics 551 **Algebra** **Fall 2006**

Week 8 Polynomial Rings
Sections 3.5-3.6

1. Show that $x^3 + x^2 + 1$ is irreducible in the ring $R = \mathbf{Z}/2\mathbf{Z}[x]$ and that $R/(x^3 + x^2 + 1)$ is a field with 8 elements.
2. Let F be a finite field with q elements.
   a) Let $f(x_1, \ldots, x_r)$ be a polynomial in $x_1, \ldots, x_r$ satisfying $f(0, \ldots, 0) = 0$ and for every $(a_1, \ldots, a_r) \neq (0, \ldots, 0)$ and $f(a_1, \ldots, a_r) \neq 0$. Show that $g(x_1, \ldots, x_r) = 1 - f(x_1, \ldots, x_r)^{q-1}$ takes value 1 at $(0, 0, \ldots, 0)$ and zero otherwise.
   b) Show that the function $g(x_1, \ldots, x_r)$ of part (a) takes the same values as the polynomial $f_0(x_1, \ldots, x_r) = (1 - x_1^{q-1}) \cdots (1 - x_r^{q-1})$, and that the degree of g is at least $r(q-1)$.
   c) Use the previous parts to prove the Artin–Chevalley theorem: A polynomial in r variables of total degree $n < r$ with coefficients in a finite field F which vanishes at $(0, \ldots, 0)$ must vanish at some nonzero point as well.
3. Hungerford 3.4.6
4. Hungerford 3.4.15
5. Hungerford 3.5.10
6. Hungerford 3.5.11
7. Hungerford 3.6.5
8. Hungerford 3.6.10