Mathematics 551 **Algebra** **Fall 2006**

Week 1 Linear algebra
Sections 7.1-7.5

1. Let R be a commutative ring or a division ring. Suppose that A, B are $n \times n$ matrices with entries from R. Show that if $AB = I_n$ then $BA = I_n$. Show that this is not true for a general ring R (using for example Hungerford exercise IV.2.13 or otherwise).
2. Let I be the ideal of $\mathbf{Z}[X]$ generated by 3 and X. Show that I is not a direct sum of cyclic $\mathbf{Z}[X]$ modules.
3. Let $S = \mathbf{R}[X]$ and let M be the direct sum of cyclic modules with annihilators $(X-1)^3, (X^2+1)^2, (X-1)(X^2+1)^2, (X+2)(X^2+1)^2$. Find the elementary divisors and invariant factors of M.
4. Show that a complex square matrix A is similar to a diagonal matrix if and only if the minimal polynomial has distinct roots.
5. Hungerford 7.2.12
6. Hungerford 7.3.3
7. Hungerford 7.4.7
8. Hungerford 7.4.10
9. Hungerford 7.5.10