Turn in starred problems Thursday 10/28/2010.

Section 7.4: 7
Section 7.5: 4*
Section 9.9: 4 (a), (d)*
Section 9.10: 2 (a), (f)*; 3

8.A* Two interacting populations $x(t), y(t)$ are described by the equations

$$x' = (3 - x - y)x \ , \qquad y' = (2 - y)y \ .$$

(a) Find all the critical points of this system. You do not need to classify these.

(b) Sketch the first quadrant $x \geq 0$, $y \geq 0$ of the phase plane, indicating, by arrows or otherwise, regions where x and y are increasing, x is increasing and y decreasing, etc., and where the trajectories are horizontal and vertical.

(c) For each initial condition below, find (from your sketch or otherwise) $\lim_{t\to\infty} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$:
(i) $x(0) = 0, \quad y(0) = 3;$ (ii) $x(0) = 3, \quad y(0) = 3;$ (iii) $x(0) = 0, \quad y(0) = 0.$

8.B* Exercise 1 from the notes on Expansions in Orthogonal Bases, available on the web page.

Comments: (a) For the problems in Section 9.10: the best approximation to a given vector within the "span" of some vectors $\{\mathbf{e}_1, \mathbf{e}_2, \ldots\}$ means the best approximation as a linear combination of those vectors.

(c) The extra exercise 8.B involves the evaluation of many integrals; these are simple (just polynomials) but it is still a nuisance. **You are welcome to use Maple, Mathematica, or some other program to do these.** If you do so, write the integral out out explicitly before giving the answer. For example, for part (a) you might write

$$\langle f, g\rangle = \int_0^2 f(x)g(x)\,dx = \int_0^2 (1-x)\,dx = 0;$$
$$\langle g, g\rangle = \int_0^2 g(x)^2\,dx = \int_0^2 (1-x)^2\,dx = \frac{2}{3}, \quad \text{so} \quad \|g\| = \sqrt{\langle g, g\rangle} = \sqrt{\frac{2}{3}}.$$