Turn in starred problems Thursday 10/21/2010.

Section 7.4: 2 (a), (b)*, (e), (k)* **See instructions below.**

Problem 7.A* Consider the equations

$$x' = x + 3y + ax(x^2 + y^2), \qquad y' = -x - y + ay(x^2 + y^2).$$

It is a fact, that you may accept as given, that the origin is the only singular point of this system.

(a) Show that for the linearized system at that point the origin is a center.

(b) For $a = 0$, $a = 1$, and $a = -1$, use Maple (or some other program) to sketch enough of the phase plane to show the behavior of trajectories near the origin. You should find that for $a \neq 0$ the origin changes to a stable or unstable focus.

(c) Is the behavior you find in (a) and (b) consistent with the Hartman-Grobman theorem? Explain.

(d) Give a qualitative discussion of how the sign of a determines the stability or unstability of the origin that you found in (b). For example: how do the a-dependennt terms in the equations affect the direction of the flow?

In the various parts of Problem 7.4.2 please do the following:

- Find all singular points;
- Obtain the *linearized system* near each singular point and classify the origin of that system (as a saddle point, unstable or stable node, unstable or stable focus, or center). If the origin is a saddle point or a node, obtain the special straight-line trajectories. Sketch the phase plane of the linearized system.
- Discuss the nature of the singularity in the true system to the extent possible; in particular, discuss whether or not you can conclude that the singularity is of the same type as the linearized system. (See the discussion on page 362 of the Hartman-Grobman theorem).
- Use Maple (or some other program) to sketch enough of the phase plane to show all the singular points and the behavior of trajectories near them. Does this confirm your analysis above (it should)? Does it answer questions that you could not answer there?
- Here is Maple code for problem 7.4.2(b); you just have to enter it. It graphs six trajectories by giving six sets of initial conditions. You should find it fairly easy to modify this code to produce the plot for other problem 7.4.2 and for the additional problem 7.A. You may want to change the ranges of the variables and to pick other initial points (you will probably need more than six trajectories to get a good picture of the behavior). Experiment until you get a nice plot.

```
with(DEtools);
phaseportrait([diff(x(t),t)=1-y(t)^2,diff(y(t),t)=1-x(t)],
    [x(t),y(t)],t=-10..10,
    [[x(0)=1,y(0)=0],
      [x(0)=1,y(0)=0.75],
      [x(0)=1.5,y(0)=1],
      [x(0)=0.5,y(0)=1],
      [x(0)=1,y(0)=1.3],
      [x(0)=1,y(0)=-0.5]],
    x=-1..3, y=-2..2,stepsize=.05,linecolor=BLUE);
```