No problems from this assignment will be collected.

Problems on the Fourier transform and the wave equation:
Section 17.10: 2, 3*, 4 (c), (f), 6 (a), (c), (g)
Section 18.4: 1, 6, 8(a,b)
Section 19.2: 5, 6, 8
Section 19.4: 4, 6 (a), (c)

Review problems:

14.A Do parts (a) through (f) of problem 6 from Section 17.8. Derive also the condition $\lambda = n^2$, $n = 1, 2, 3 \ldots$, and the values of the first few Chebyshev polynomials, by a series solution of the differential equation:

(a) with center at the ordinary point $x_0 = 0$;

(b) with center at the singular point $x_0 = 1$.

14.B Consider the following problem for the function $u(x, t)$:

$$4\,u_{xx} = u_t, \qquad 0 < x < 3, \quad t > 0; \tag{14.1}$$

$$u(0, t) + a u_x(0, t) = 0, \qquad u(3, t) = 0, \qquad t > 0; \tag{14.2}$$

$$u(x, 0) = 1, \qquad 0 < x < 3. \tag{14.3}$$

(a) Separate variables and investigate the eigenvalues of the resulting Sturm-Liouville problem. In particular, show that (i) if $a > 3$ or $a \le 0$ then all eigenvalues are positive, (ii) if $a = 3$ then zero is an eigenvalue and all other eigenvalues are positive, and (iii) if $0 < a < 3$ then there is one negative eigenvalue and all other eigenvalues are positive. You will not be able to find the eigenvalues analytically (except when $a = 0$).

(b) In each case above, find the solution of the problem as an infinite series. Express the coefficients as ratios of integrals, but do not attempt to evaluate them. The series and integrals will involve the eigenvalues from (a), so you won't be able to be too specific.

(c) Discuss the behavior of $u(x, t)$ as $t \to \infty$. You should find, in the various cases of (a), that (i) $u(x, t)$ approaches zero as $t \to \infty$; (ii) $u(x, t)$ approaches a non-zero steady state as $t \to \infty$; (iii) $u(x, t)$ becomes infinite ("blows up") as $t \to \infty$.

(d) What is the physical interpretation of the boundary condition at $x = L$ when $a > 3$, and why, on physical grounds, does the solution blow up in that case?

Comments, hints, instructions: 1. Hint for 17.10:3: Treat separately the integrals over $x > 0$ and $x < 0$.

2. In 18.4:8(b) one should use the Laplace transform (in the variable t), not the Fourier transform, although in fact one can guess the form of the solution, and then find it completely, by elementary reasoning.

3. 14.A and 14.B are rather long, and you don't have to do them, but I think that they provide a good review of material from several parts of the course. Note: when applying the Frobenious method in 14.A (center $x_0 = 1$) you should be able to see immediately that one of the two solutions does not satisfy the given boundary conditions.