Turn in starred problems Thursday 12/2/2010.

Section 18.3: 6 (f)*, (i), (l), (m), 10 (a), (c)*, (e), (j)*, 14, 15, 19, 29*

13.A* Here is a variant of the periodic boundary condition problem of Section 17.8:
Find the eigenvalues and eigenfunctions for

$$y'' + \lambda y = 0, \quad y(0) = -y(1), \quad y'(0) = -y'(1).$$

Find also the eigenfunction expansion of $f(x) = 1$.

13.B* A metal plate has the form of a quarter of a disk; it fills the region described in polar coordinates by $0 \le r \le a$, $0 \le \theta \le \pi/2$. Heat conduction in this plate, with Dirichlet boundary conditions on all edges, is described by the equations:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}, \qquad 0 \le r \le a, \quad 0 \le \theta \le \pi/2, \quad t > 0;$$
$$u(a,\theta,t) = 0, \qquad 0 \le \theta \le \pi/2, \quad t > 0;$$
$$u(r,0,t) = u(r,\pi/2,t) = 0, \qquad 0 \le r, \quad t > 0;$$

Use separation of variables to find all product solutions $u(r,\theta,t) = R(r)\Theta(\theta)T(t)$ of these equations. Follow the model of the solution for the full disk in the posted notes. Note that you are not asked to form superpositions of these product solutions or to solve any initial value problem.

1. Problems 14 and 29 of Section 18.3 give two different approaches to the same problem. You may find it useful to look at both of them.

2. Problem 18.3:19. I find the language of the text somewhat confusing. In the language I have used in class, I would say that:

- One first introduces the particular solution $z(x) = (x-L)^2/2L$, writes $v(x,t) = u(x,t) - z(x)$, and determines what problem $v(x,t)$ satisfies;
- For the v problem one introduces another particular solution $v_2(t)$, reducing to a problem for $v_1(x,t) = v(x,t) - v_2(t)$ which one knows how to solve.

3. In 12.B, separation of variables should lead to a problem for $\Theta(\theta)$ which is familiar; you don't have to rederive its solution. In solving the Sturm-Liouville problem for $R(r)$ you may as needed make convenient assumptions about the sign of the eigenvalues, as in the notes.