No problems from this assignment will be collected.

Section 19.2: 5, 6, 8

Section 19.4: 4, 6 (a), (c)

13.A Consider the following problem for the function $u(x,t)$:

$$9\,u_{xx} = u_t, \qquad 0 < x < 1, \quad t > 0; \tag{1.1}$$

$$u(0,t) = 0, \qquad \gamma u(1,t) + u_x(1,t) = 0, \qquad t > 0; \tag{1.2}$$

$$u(x,0) = 1, \qquad 0 < x < 1. \tag{1.3}$$

(a) Separate variables and investigate the eigenvalues of the resulting Sturm-Liouville problem. In particular, show that (i) if $\gamma > -1$ then all eigenvalues are positive, (ii) if $\gamma = -1$ then zero is an eigenvalue and all other eigenvalues are positive, and (iii) if $\gamma < -1$ then there is one negative eigenvalue and all other eigenvalues are positive. You will not be able to find the eigenvalues analytically. NOTE: We discussed the case $\gamma = -1$ in class, and Example 3 of Section 17.7 of our text is a model for the case $\gamma > -1$ (although in the text example the Robin boundary condition is imposed at the left, not the right, end of the rod).

(b) In each case above, find the solution of the problem as an infinite series. Express the coefficients as ratios of integrals, but do not attempt to evaluate them. The series and integrals will involve the eigenvalues from (a), so you won't be able to be too specific.

(c) Discuss the behavior of $u(x,t)$ as $t \to \infty$. You should find, in the various cases of (a), that (i) $u(x,t)$ approaches zero as $t \to \infty$; (ii) $u(x,t)$ approaches a non-zero steady state as $t \to \infty$; (iii) $u(x,t)$ becomes infinite ("blows up") as $t \to \infty$.

(d) What is the physical interpretation of the boundary condition at $x = L$ when $\gamma > 1$, and why, on physical grounds, does the solution blow up in that case?