Suggested problems for 11/14/06[1]

Minima and Maxima

1. Over all sequences of positive integers that sum to 1000, determine the one whose product is maximum.

2. A collection of tennis players play a tournament in which each pair of players plays one match. A player p is called *weakly dominant* if for every other player q, p beat q or p beat at least one player that beat q. Prove that there is at least one weakly dominant player.

3. If S is a set of real numbers, let $A(S)$ be the set of numbers of the form $(x+y)/2$ where $x, y \in S$. For each positive integer n determine the minimum size of $A(S)$ over all sets S of size n.

4. Suppose that f is a continuous function on $[0,1]$ such that for each $j \in \{0, 1, \dots, n-1\}$

$$\int_0^1 x^j f(x) = 0.$$

and

$$\int_0^1 x^n f(x) = 1.$$

Prove that the maximum of $|f(x)|$ on $[0,1]$ is greater than $2^n(n+1)$.

5. Putnam 1998:B1) Find the minimum value of:

$$\frac{(x+1/x)^6 - (x^6+1/x^6) - 2}{(x+1/x)^3 + (x^3+1/x^3)},$$

for $x > 0$.

6. (Putnam 1993:B1) Find the smallest positive integer n such that for every integer m with $o < m < 1993$, there is an integer k for which $m/1993 < k/n < (m+1)/1993$.

7. (Putnam 1988:B3) For every n in the set $\mathbb{N}$ of positive integers, let r_n be the minimum of $|c - d\sqrt{3}|$ over all nonnegative integers c and d with $c + d = n$. Find, with proof, the lease positive real number y such that $r_n \leq y$ for all $n \in \mathbb{N}$.

8. Putnam 1991:B6. Let a, b be positive numbers. Find the largest number c in terms of a and b such that

$$a^x b^{1-x} \leq a\frac{\sinh ux}{\sinh u} + b\frac{\sinh u(1-x)}{\sinh u}$$

, for all u satisfying $0 \leq |u| \leq c$ and for all x, $0 < x < 1$.

[1] Version:11/7/06