Math 152 Problems, Week 11

1. For each of the following series $\sum_{n=1}^{\infty} a_n$, find a formula for a_{n+1}/a_n and the limit of this ratio as $n \to \infty$. Then use the ratio test to find the radius of convergence of each of the following power series. Also test the endpoints to find the interval of convergence for each series.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n(3x)^n}{(n+1)(n+2)}$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3(x-1)^n}{(2n+1)!}$ c) $\displaystyle\sum_{n=1}^{\infty} \sqrt{n}(3x+2)^n$

2. Define the *Rutgers function* $R(x) = 1766 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots = 1766 + \displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n!}$

a) Find the radius of convergence of this series.

b) Differentiate the series for $R(x)$ term-by-term to show that $R'(x) = R(x) - 1765$.

c) Solve the differential equation $y' = y - 1765$ with the initial condition $y(0) = 1766$. Use this to express $R(x)$ in terms of e^x.

3. In this problem, $f(x) = \dfrac{x^2}{16 + x^4}$

a) Use the formula for the sum of a geometric series to express $f(x)$ as a power series in x. What is the radius of convergence of this series?

b) Integrate the power series in a) term-by-term to find a power series for the indefinite integral $\displaystyle\int f(x)\,dx$

c) Use the series in b) to express $\displaystyle\int_0^1 \frac{x^2\,dx}{16+x^4}$ as a convergent infinite series.

d) Use the alternating series remainder estimate to determine how many terms of the series in c) are needed to approximate the definite integral to 5 decimal places (this means that the error must be less than 5×10^{-6}). Then calculate the partial sum of the series with this number of terms, and compare with the approximate value from the `fnint(` program.

4. Define a function $f(x)$ by the power series

$$\sum_{n=0}^{\infty} (n+2)(n+1)x^n = 2 \cdot 1 + (3 \cdot 2)x + (4 \cdot 3)x^2 + (5 \cdot 4)x^3 + \cdots$$

a) Let $a_n = (n+2)(n+1)x^n$. Find a formula for a_{n+1}/a_n and the limit of this ratio as $n \to \infty$. Now use the ratio test to find the radius of convergence of this series.

b) Define $g(x) = \displaystyle\int_0^x f(t)\,dt$, where $f(x)$ is the function defined above. Use term-by-term integration to find the power series for $g(x)$.

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c) Let $h(x) = \int_0^x g(t)\,dt$, where $g(x)$ is the function defined in b). Use term-by-term integration to find the power series for $h(x)$.

d) Use the formula for the sum of a geometric series to obtain a formula for $h(x)$. Now differentiate $h(x)$ twice to find an expression for $f(x)$ as a rational function.

5. Express the functions given by the following series in terms of the functions on your calculator:

a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n(n+1)!}$ b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$ c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$