Math 152 Problems, Week 8

Calculators and Series: The TI-82 and TI-85 have several built-in functions that are useful for these problems*.

Some Warm-Up Exercises:

A) Create and store a list whose entries are $\{3^2, 5^2, \ldots, 11^2\}$.

TI-82: Go to `LIST` (the alternate to `STAT`) and select `seq(`.

TI-85: Go to `MATH MISC` and select `seq(`.

Fill in the arguments in the `seq(` function as $\texttt{seq(}N^2, N, 3, 11, 2)$ and press `ENTER`. Here N is the ordinary *alphabetic variable*; it goes from $N = 3$ to $N = 11$ in steps of 2.
Check that the displayed list is correct. Now store this list of numbers so that you can make calculations on it later.

TI-82: press `STO` L_1 to store this list as the variable L_1 (the special List variables L_1, L_2 ... are the 2nd keys of 1, 2, ...). You can view the list by pressing L_1.

TI-85: press `STO` L (the alphabetic variable) to store this list with the name L (you can choose any name). You can view the list by `RCL` L.

B) Calculate $3^2 + 5^2 + \ldots + 11^2$ in two ways: First by adding terms, and then by the following operation.

TI-82: Go to `LIST MATH`, select `sum`, and press L_1.

TI-85: Go to `MATH MISC`, select `sum`, and enter L.

You can also do arithmetic on lists. For example, if you have two series with terms a_n and b_n stored as lists L_1 and L_2 on the TI-82 (or as L and M on the TI-85), then the expression $3L_1 + 4L_2^5$ (or $3L + 4M^5$) will give the list with terms $3a_n + 4b_n^5$.

C) Create and store a list with terms $\{1, 2, 3, 4, 5\}$. Then use operations on this list to calculate $3^2 + 5^2 + \ldots + 11^2 = \sum_{k=1}^{5}(2k+1)^2$ once again.

1. Let $a_n = \dfrac{5^{2n}}{7^{3n+1}}$ for $n = 0, 1, 2, \ldots$ and let $s_k = \sum_{n=0}^{k} a_n$.

a) Write out the expressions for s_1, s_2, and s_3 as sums of rational numbers *without* using your calculator. You do not have to carry out the arithmetic.

b) Create a list whose entries are $\{a_0, a_1, a_2, \ldots, a_{20}\}$ and calculate $\sum_{n=0}^{20} a_n$.

c) Use the formula for the sum of a geometric series to find $\sum_{n=0}^{\infty} a_n$. Compare with the numerical value of the partial sum in c).

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* Check the manuals for more details. Other calculators have equivalent capabilities.

2. If $f(x)$ is a positive, decreasing function defined for $N \le x < \infty$, then

$$(*) \qquad \int_{N+1}^{\infty} f(x)\,dx \le \sum_{n=N+1}^{\infty} f(n) \le \int_{N}^{\infty} f(x)\,dx.$$

(see Figures 1 and 2 on p. 599 of the text). These inequalities prove the **Integral Test**:

The series $\sum_{n=N}^{\infty} f(n)$ converges if and only if the improper integral $\int_{N}^{\infty} f(x)\,dx$ converges.

Use the integral test to determine which of the following series converge and which diverge. For the series that converge, calculate the sum of the terms up to $n = 10$ (see calculator instructions above) and obtain upper and lower bounds for the remainder R_{10} by using $(*)$ with $N = 10$.

a) $\sum_{n=1}^{\infty} n\, e^{-n^2}$ b) $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$ c) $\sum_{k=1}^{\infty} \frac{k}{(1+k^2)^3}$

3. Let $a_n = \frac{6}{(n+2)n}$ for $n = 1, 2, \ldots$ and let $s_k = \sum_{n=1}^{k} a_n$ be the kth partial sum of the series with terms a_n.

a) Write out s_2, s_3 and s_4 as sums of rational numbers *without* using your calculator (don't carry out the arithmetic)

b) Create a list whose entries are $\{a_1, a_2, \ldots, a_{50}\}$ and sum the entries in this list to calculate s_{50}.

c) Use the partial fractions identity $\frac{6}{(n+2)n} = \frac{3}{n} - \frac{3}{n+2}$ to calculate s_3, s_4, and s_5. Do you see any pattern? Find an *explicit* formula for s_n as a function of n and use it to calculate $\lim_{n\to\infty} s_n$. Compare with the numerical value from b).