# Math 152 Problems, Week 4

1. Use appropriate trig substitutions to evaluate:

a) $\int \sqrt{4-x^2}\,dx$.

b) $\int \sqrt{4+x^2}\,dx$

c) $\int \sqrt{5+2x+x^2}\,dx$

2. The region $R$ is bounded below by the $x$-axis, bounded on the left by the line $x = 2$, bounded on the right by the line $x = 3$, and bounded above by the curve

$$y = \frac{1}{x^2+3x+2}$$

a) Sketch the region $R$ and set up a definite integral that gives the area of $R$. Then calculate the integral in two ways:
(i) the `fnInt(` program in your calculator
(ii) method of partial fractions

b) The region $R$ is rotated around the $x$-axis to generate a solid body $B$. Sketch $B$ and set up a definite integral that gives the volume of $B$. Calculate the integral in two ways:
(i) the `fnInt(` program in your calculator
(ii) method of partial fractions (be careful how you set this up–there are *four* undetermined coefficients in the partial fraction decomposition).

3. Evaluate $\int \sin x \cos x\,dx$ using three *different* methods:

a) substitution b) trig identity c) integration by parts

Now show that all three methods give the same answer.

4. Many integrals can be done with "rationalizing substitutions" which change the integrals into integrals involving rational functions. In turn, these integrals can be computed (theoretically) using partial fractions. Here are two examples of such integrals:

a) Integrate:

$$\int \frac{1}{(\sqrt{x}+1)(\sqrt{x}+2)}\,dx$$

Try $x = t^2$.

b) Integrate:

$$\int \frac{e^x+1}{e^{2x}+1}\,dx$$

Try $t = e^x$.

**(Over)**

5. A water storage tank has the shape of a cylinder with diameter 10 ft. It is mounted so the axis of the cylinder is horizontal (the circular cross-sections of the cylinder are vertical). If the depth of the water is 7 ft., what percentage of the total capacity of the tank is filled?

Draw a picture and set up the problem. Then solve it in three (3) ways:

a) By elementary geometry (compare areas of circular sectors)

b) Express the answer in terms of a definite integral, and then obtain an approximate numerical value for the integral using the `fnInt(` function on your calculator.

c) Evaluate the integral in b) *exactly* in terms of elementary functions using a trig substitution, and then obtain approximate numerical values for these functions using your calculator.

Of course, all three methods should give the same answer!