Math 152 Problems, Week 3

1. Evaluate each indefinite integral using substitutions and integration by parts, as needed:

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx \qquad \int x^3 e^{x^2}\,dx$$

$$\int \frac{\cos\left[\ln\left(x^2\right)\right]}{x}\,dx \qquad \int x\arctan x\,dx$$

2. The outdoor temperature (in $^\circ$ F) on a particular day was approximated by the function

$$T(t) = 50 + 14\sin\frac{\pi t}{12},$$

where t is time (in hours) after 9 am.

a) Find the *maximum* temperature $T_{\max}$, *minimum* temperature $T_{\min}$, and *average* temperature

$$T_{\text{aver}} = \frac{1}{12}\int_0^{12} T(t)\,dt$$

on that day during the period between 9 am and 9 pm.

b) At what times t during the day was $T(t) = T_{\text{aver}}$? (The Mean Value Theorem for integrals says that at least one such time must exist.)

c) Show that $T_{\text{aver}} \neq \frac{1}{2}(T_{\min} + T_{\max})$ (This is the definition that the weather bureau uses for "average temperature").

d) Suppose that $T(t)$ is a *linear* function of t. Use the graph of $y = T(t)$ in this case to show that $T_{\text{aver}} = \frac{1}{2}(T_{\min} + T_{\max})$.

3. The following information is known about a function T and its derivatives:

x	$T(x)$	$T'(x)$	$T''(x)$
1	2	-2	2
2	3	6	5
3	7	4	-4
4	2	5	7

Compute the following integrals:

a) $\displaystyle\int_2^3 T'(x)\,dx$ b) $\displaystyle\int_2^3 T''(x)\,dx$

c) $\displaystyle\int_2^3 xT''(x)\,dx$ d) $\displaystyle\int_2^3 x^2T'''(x)\,dx$

(Over)

4. If a freely falling body starts from rest, then its velocity is given by $v(t) = gt$ and its displacement is given by $s = \dfrac{1}{2}gt^2$.

a) Suppose the body starts at rest and falls freely a distance of 1,000 feet. Calculate the time T (seconds) that this takes ($g = 32\frac{\text{ft}}{\text{sec}^2}$) and the *time average* of the velocity of the body

$$v_{\text{time aver}} = \frac{1}{T}\int_0^T v(t)\,dt.$$

Draw a graph of the function $v(t)$ for $0 \le t \le T$. Find the time t when $v(t) = v_{\text{time aver}}$ and give a graphical interpretation.

b) Find a formula for the velocity as a function $f(s)$ of displacement s, and calculate the *distance average* of the velocity of the body in situation a):

$$v_{\text{dist aver}} = \frac{1}{1000}\int_0^{1000} f(s)\,ds.$$

(notice that $v_{\text{dist aver}} \neq v_{\text{time aver}}$). Draw a graph of the function $v = f(s)$ for $0 \le s \le 1000$. Find the distance s that the body has fallen when $f(s) = v_{\text{dist aver}}$ and give a graphical interpretation.

5. In many problems in physics and engineering, the *second derivative* plays an important role (because of Newton's Law $F = ma$). Let $f(x)$ and $g(x)$ be differentiable functions. Use integration by parts to express

$$\int_0^1 f''(x)g(x)\,dx - \int_0^1 f(x)g''(x)\,dx$$

in terms of $f(0), f'(0), f(1), f'(1), g(0), g'(0), g(1), g'(1)$.