Math 152 Problems, Week 1

1. (a) Evaluate the indefinite integral

$$\int \frac{x^2}{\sqrt{2+x^3}}\,dx$$

using the substitution $u = 2 + x^3$. Remember to substitute the appropriate expression for dx (in terms of u and du).

(b) Find appropriate substitutions to evaluate the following indefinite integrals:

$$\int e^x(1+e^x)^{27}\,dx \qquad \int \frac{x}{1+x^4}\,dx$$

$$\int x^2\cos(1-x^3)\,dx \qquad \int \frac{\sin x}{1+\cos^2 x}\,dx$$

(c) Suppose f is continuous and $\int_0^9 f(u)\,du = 4$. Find $\int_0^3 x f(x^2)\,dx$. (*Hint:* Try a substitution in the integral.)

(d) Use a substitution to prove that

$$\int_0^1 x(1-x)^{1766}\,dx = \int_0^1 x^{1766}(1-x)\,dx.$$

Then evaluate the definite integral.

2. The function

$$C(x) = \int_0^x \cos(\pi t^2/2)\,dt$$

is used in the theory of diffraction of light waves.

(a) Find $C'(x)$ and plot its graph for $0 \le x \le 2$. On what intervals is $C(x)$ increasing?

(b) Find $C''(x)$ and plot its graph for $0 \le x \le 2$. On what intervals is $C(x)$ concave upwards?

(c) Use the graph in (a) to show that $1/2 < C(1) < 1$. Then use the `fnInt(` program on your calculator to find a more exact value for $C(1)$.

3. Suppose f is a continuous function. Prove that

$$\int_0^3 f(x+2)\,dx = \int_2^5 f(t)\,dt.$$

Give two proofs: one by substitution, and another by geometry (interpret each integral geometrically as a signed area).

4. a) Sketch the region $0 \le y \le 1/x^2$, $1 \le x \le 4$.

b) Find the number a such that the line $x = a$ divides the region in a) into two parts of equal area.

c) Find the number b such that the line $y = b$ divides the region in a) into two parts of equal area.