MATH 151–04,05,06 EXAM #2: November 23, 1998Name: ______________________

INSTRUCTIONS: Do all problems.

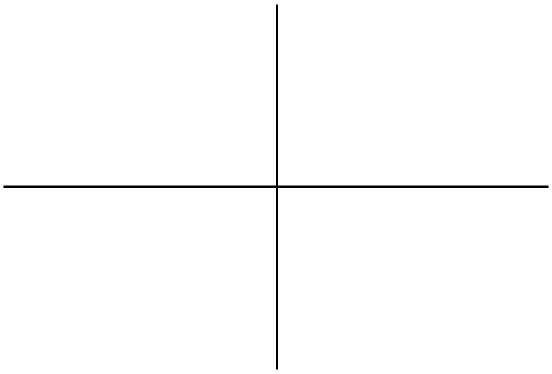
Show all your work. Full credit may not be given for an answer alone.

You may use a TI-82 calculator or equivalent.
You may not use a TI-92 or any calculator with a QWERTY keyboard.
You may have one 8.5×11 **handwritten sheet of formulas.**

(12) 1.

Let $f(x) = \dfrac{x^4}{80} + \dfrac{x^3}{12} - 2.$

Get a calculator sketch of the graph of f in the window $-10 \le x \le 10$, $-10 \le y \le 10$ and copy it roughly here:



(a) Algebraically, find the intervals where $f'(x)$ is positive, negative, or zero, and explain how this fits with your graph. **You must show your calculations.**

(b) Do the same with $f''(x)$, again explaining how it fits with your graph.

(c) Give the precise x-coordinates of all local maxima, local minima and points of inflection, even if they aren't included in the part of the graph shown above.

(16) 2. In this problem you will sketch the graph of a function f. The formula for f is unavailable to you, but you are given its **derivative**:

$$f'(x) = e^{2x-x^2} - 1.$$

(Do not attempt to find an explicit formula for $f(x)$; you won't succeed.) You are also given that $f(0) = -4$.
By analyzing the derivative, etc., sketch the graph of the function f, showing (and labelling) local maxima and minima, inflection points and intervals where f is increasing, decreasing, concave up, and concave down.

(12) 3. (a) Find the linear approximation to the function $f(x) = \dfrac{1}{\sqrt{x}}$ at the point $a = 25$.

(b) What approximate value does this give you for $\dfrac{1}{\sqrt{25.3}}$?

(c) Is this approximate value an overestimate or underestimate of the true value? Explain, in terms of the first or second derivative of f and using the graph of f.

(12) 4. Suppose that the `ln` key on your calculator were broken and you decided to compute $\ln 2$ by applying Newton's method to the equation

$$f(x) = e^x - 2 = 0, \text{ starting with initial guess } x_0 = 0.$$

(a) Find the next approximations x_1 and x_2. Do not give decimal values, but give exact answers in terms of $e, 1, 2$, etc.

(b) Without actually calculating x_3, show in a diagram how x_3 arises from x_2 and decide from that diagram whether x_3 will be larger or smaller than x_2. Use the actual graph of f, and explain what the first and second derivative of f have to do with your decision. (Label the diagram clearly!)

(12) 5. Ant A and ant B are carrying a stick whose length is 5 cm. Ant A is at one end of the stick and is crawling up the positive y-axis. Ant B is at the other end and is crawling on the positive x-axis (toward the origin). Ant A travels at a constant rate of 0.5 cm/sec.

(a) At the moment when ant A is 2 cm from the origin, how fast is ant B going?

(b) At the same moment, is the area of the triangle formed by the two axes and the stick increasing or decreasing, and at what rate?

(c) At the same moment, how fast is the angle between the stick and the x-axis changing?

(12) 6. Assume that Newton's Law of Cooling applies in this problem. In a room which is at $20°C$, assume that the temperature of a cup of coffee was 80° a minute ago and is 75° now.

(a) When will it be 50°?

(b) What was its temperature 3 minutes ago?

(24) 7. Find the following (exact answers!):

(a) $\displaystyle\lim_{x\to 0^+} \frac{\arctan(1/x)(2^x-1)}{\sin 3x}$. (Note: For 2 points less, you may do $\displaystyle\lim_{x\to 0^+} \frac{\arctan(1/x)(e^x-1)}{\sin 3x}$ instead.)

(b) The domain of the function $f(x) = \arcsin(3-7x)$.

(c) $\displaystyle\lim_{x\to e^2} \ln(x^3)$.

(d) An equation of the tangent line to the graph of $y = \arcsin(\ln(5x+1))$ at the origin.

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