Math 151, Spring 2009, Review Problems for Exam 1

Your first exam is likely to have problems that do not resemble these review problems.

PRECALCULUS

1. Find the domain and range of the following functions.
   (a) $\sqrt{3-x}$ (b) $\frac{1}{\sqrt{x^2+1}}$ (c) $\dfrac{1}{\sqrt{2-x}}$

2. Express the set of real numbers x satisfying the given condition as an interval.
   (a) $|x+2|<7$ (b) $|3x-1|\geq 5$ (c) $|x+2|<7$ and $|3x-1|\geq 5$ are true.

3. Find $(f\circ g)(x)$ and $(g\circ f)(x)$, where $f(x)=\sqrt{x^2+2}$ and $g(x)=x^2+1$.

4. Find the inverse of the function $f(x)=\dfrac{x}{x+1}$.

5. Simplify $\cot(\sin^{-1}(x))$ and $\sin^{-1}\left(\sin\left(\dfrac{4\pi}{3}\right)\right)$.

LIMITS AND CONTINUITY

6. Find the following limits
   (a) $\displaystyle\lim_{x\to 2}\frac{\sqrt{x+1}-1}{x+3}$ (b) $\displaystyle\lim_{x\to 3}\frac{x(x+1)}{x+3}$

7. Find the following limits
   (a) $\displaystyle\lim_{x\to 2}\frac{\sqrt{4x+1}-3}{x-2}$ (b) $\displaystyle\lim_{x\to 5}\frac{x(x-5)}{\sqrt{x}-\sqrt{5}}$ (c) $\displaystyle\lim_{x\to 2}\frac{x^2+4x-12}{x^2-9x+14}$

8. Show that the equation $x^2-\cos(x)=1$ has a solution in the open interval $(1,2)$.

9. Use the Squeeze Theorem to show that $\displaystyle\lim_{x\to 0}x^2\sin\left(\frac{3}{x^3}\right)=0$.

10. Find the following limits
    (a) $\displaystyle\lim_{x\to 0}\frac{\tan(3x^2)}{x^2}$ (b) $\displaystyle\lim_{x\to 0}\frac{\cos^2(x)-\cos(x)}{x}$ (c) $\displaystyle\lim_{x\to 0}\frac{\sin(4x)}{\sin(7x)}$
    (d) $\displaystyle\lim_{x\to 0}\frac{2x+2\sin(x)+6\cos(x)-6}{3x}$

11. True or false? The following limit exists

$$\lim_{x\to 5}\frac{|x-5|}{x-5}.$$

    Give details to justify your answer.

12. True or false? The function

$$f(x) = \begin{cases} 2x^2 + 1, & x > 3 \\ 19, & x = 3 \\ 5x + 4 + \sin(x - 3), & x < 3 \end{cases}$$

is continuous at $x = 3$. Give details to justify your answer.

13. Find the following limits
(a) $\lim_{x \to 4^-} \frac{4 - x}{|x - 4|}$ (b) $\lim_{x \to 3^+} \frac{|x - 3|}{x - 3}$

14. Find the values of a and b that will make the function

$$f(x) = \begin{cases} x^2 + 1, & x < 1 \\ ax + b, & 1 \leq x \leq 2 \\ x^3, & x > 2 \end{cases}$$

continuous everywhere.

15. Use the $\epsilon - \delta$ formal definition of the limit to prove that $\lim_{x \to 2} 6x + 2 = 14$.

Derivatives

16. Use the limit definition of the derivative to compute $f'(x)$ for $f(x) = 2x^2 + 1$.

17. Do the following:
(a) Find the equation for the tangent line to the curve $y = x^3 + x^2 + x + 1$ at the point $(1, 4)$.
(b) Find the equation for the line that also passes through $(1, 4)$, but is perpendicular to the line you found in (a).

18. Let $f(x) = x + \frac{1}{x}$. Find all the points on the graph $f(x)$ where the tangent line is horizontal.

19. Find the derivative of each function.
(a) $(2x + 1)^3 e^{2x}$ (b) $\frac{x^2 + x + 1}{\sin(2x)}$ (c) $\tan(x^3 + 3x + 1)$

20. Find the second derivative of each function.
(a) $(x^2 + 1)^{20}$ (b) $\frac{x}{x + 1}$ (c) xe^{x^2}

21. An object is moving along the x-axis and its position at any time $t \geq 0$ is given by $x(t) = -2t^3 + 3t$. Find the velocity and acceleration of the object at $t = 1$. Is the object moving forward or backwards at $t = 1$? Is it speeding up or slowing down at $t = 1$?

22. Assume that $f(2) = 3, f'(2) = -1, g(2) = -2, g'(2) = 6, f'(-2) = -2$, and $g'(3) = 4$. Use this information to calculate $(f \circ g)'(2)$ and $(g \circ f)'(2)$.