Math. 138 Review Problems, S2008

1. Evaluate the following integrals.

(a) $\int_e^{10} x^6 \ln x dx$ (b) $\int \cos x e^x dx$ (c) $\int \frac{x}{x^2-9} dx$

2. Let $I = \int_1^3 e^{x^2} dx$

(a) Use the Trapezoidal Methods with $n = 4$ to approximate I .

(b) Estimate the error.

3. Check if each one of the following improper integrals divergent or convergent. Show all work: evaluate the integral and take the appropriate limits.

(a) $\int_0^{\infty} \frac{\ln x}{x} dx$ (b) $\int_0^{\infty} x e^{-x} dx$

4. (a) Find the 4th Taylor polynomial of $f(x) = e^{2x}$ about 1.
(b) Estimate the reminder, $|R_n(x)|$ for $x = 1.002$.

5. Solve each of the following differential equations.

(a) $\frac{dy}{dx} = 1 + x - y^2 - xy^2 \qquad y(0) = 4$

(b) $x\frac{dy}{dx} + y = xe^{2x} \qquad x = 1, y = 1$

(c) $y'''' + 4y'' - 5 = 0$

6. Use the method of undetermined coefficient and variation of variable to find the general solution of the non homogenous second order differential equation $y'' - 3y' - 4y = 2\sin x$.

7. Let $A = \begin{pmatrix} -2 & 2 & 1 \\ 1 & 5 & 1 \\ 6 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 4 \\ 1 & 5 & -7 \end{pmatrix}$

Evaluate the matrix $(2B - A)B$.

8. For the matrix $B = \begin{pmatrix} 4 & 1 & -3 \\ 0 & 0 & 2 \\ 0 & 0 & -3 \end{pmatrix}$, find all its eigenvalues and the matching eigenvectors.

9. Let $\begin{cases} x + y - z = 1 \\ 3x - 2y + z = 3 \\ 4x + y - 2z = 0 \end{cases}$ be a system of linear equations.

Solve it using all the following methods:

(a) Elementary row operation on the augmented matrix of the system.

(b) Inverse matrix.

(c) Cramer's rule .

10. Real life problems

(a) The population density of Hope City is $D(r) = 100r^2 - 5r + 250$ where r is the distance from the center of the city. How many people are living between 2 to 3 miles form the center of the city?

(b) The number of cells in a growing dish doubles every 3 hours.
(i) Write the differential equation that describes the rate of growth and solve it.
(ii) How many cells the are after 10 hours?
(iii) After how many hours the number of cells will multiply by 7?

(c) A population $P(t)$, satisfying the logistics equation values $P(1970) = 150$, $P(1980) = 200$ *and* $P(1990) = 220$ in thousands of individuals. Find the equation for the population, $P(t)$, at any time t.