Sims 2006 Exam 2, no solutions

1A

(6) 1. Compute the derivatives of the following functions:

a) $xe^{\cos x}$

b) $\tan^3(x^3)$

(8) 2. Compute the following limits:

a) $\lim_{x \to 0} \dfrac{e^{5x} - 5x - 1}{x^2}$

b) $\lim_{x \to 1} \dfrac{x^2 + 1}{x + 1}$

(13) 3. Find an equation for the line tangent to the graph of $\ln y + x^3 + 2xy = 10$ at the point $(2, 1)$.

(8) 4. A certain function $f(x)$ is defined and differentiable for all real numbers x. If $f(1) = 2$ and $|f'(x)| \leq 3$ for $1 < x < 3$, what is the largest possible value of $f(3)$? What is the smallest possible value of $f(3)$? Give brief explanations of your answers.

(10) 5. Below is a portion of the graph of a function f.

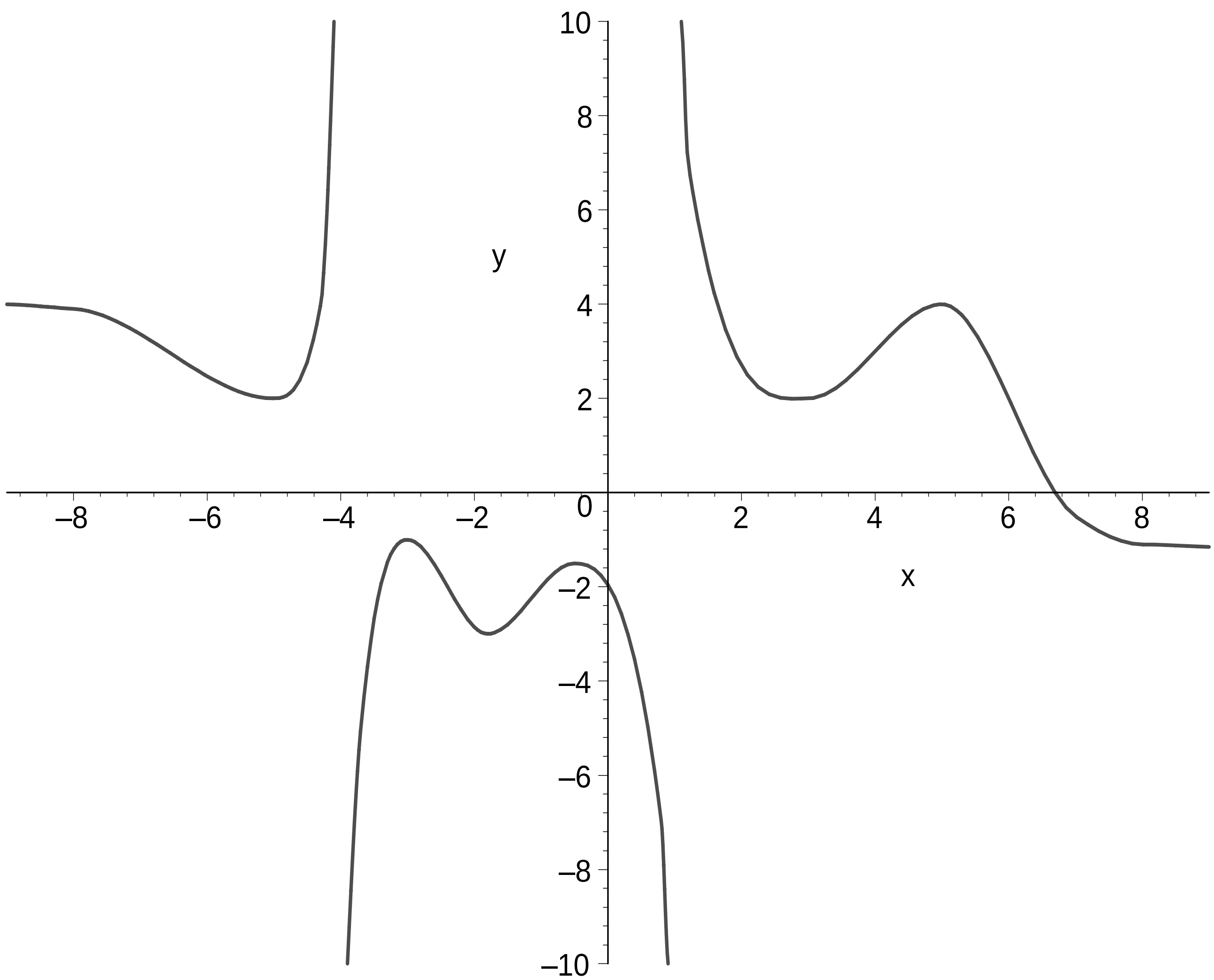
a) On the plot, draw lines that appear to be vertical asymptotes of the graph. Label each of the lines with the letter V.

b) On the plot, draw lines that appear to be horizontal asymptotes of the graph. Label each of the lines with the letter H.

c) On the graph of the function, place a small dot at each place the function has a relative maximum. Label each of these points with the letter A.

d) On the graph of the function, place a small dot at each place the function has a relative minimum. Label each of these points with the letter Z.

e) On the graph of the function, place a small dot at each place the function has a point of inflection. Label each of these points with the letter I.



(15) 6. What are the absolute maximum and the absolute minimum of the function $x^3 - 3x^2 + 7$ on the interval $[1, 4]$?

(10) 7. Suppose that $f(x) = e^{3x^2-3}$.

Compute $f(1)$.

Compute $f'(1)$.

Use the linearization (differential, tangent line approximation) of f at $x = 1$ to estimate $f(1.05)$.

(15) 8. For some mysterious reason the dimensions of a rectangular box are changing. At a certain moment, the length is increasing at a rate of 2 feet per hour, the width is decreasing at a rate of 3 feet per hour, and the height is increasing at a rate of 4 feet per hour. If at that moment the length is 5 feet, the width is 6 feet, and the height is 3 feet, how fast is the volume of the box changing? (Be sure to give the units.) Is the volume increasing or decreasing? (Note: The volume of a rectangular box is the product of the length, the width, and the height.)

(15) 9. A manufacturer can produce shoes at a cost of \$50 a pair and estimates that if the shoes are sold for p dollars a pair, then consumers will buy approximately

$$1000e^{-0.1p}$$

pairs of shoes each week. At what price should the manufacturer sell the shoes to maximize profits?