(13) 1. Find an equation for the tangent line to the graph of $y^2 = x^3 - 3xy + 3$ at the point $(-2, 1)$.

(10) 2. Find equations for all vertical and horizontal asymptotes of the function

$$f(x) = \frac{3e^x + 5}{7e^x - 2}.$$

(All numbers used should be described by exact expressions, not decimal approximations. Thus you should write $\sqrt{2}$, not 1.414.)

(15) 3. At a certain time, the length of a rectangle is 5 feet and its width is 3 feet. At that same moment, the length is *decreasing* at 0.5 feet per second and the width is *increasing* at 0.4 feet per second.

What is the length of the diagonal at that time?

How fast is the length of the diagonal changing? Is this length increasing or decreasing?

(10) 4. Suppose that $f(x) = \sqrt{2 + 7x^3}$.

Compute $f(1)$.

Compute $f'(1)$.

Use the linearization (differential, tangent line approximation) of f at $x = 1$ to estimate $f(1.08)$.

(5) 5. A friend runs up to you and excitedly explains that she has found a function g with the following properties:

g is continuous on $[0, 1]$ and differentiable on $(0, 1)$.
$g(0) = 1$ and $g(1) = 5$.
$g'(x) \leq 3$ for all x in $(0, 1)$.

Explain why you doubt your friend's claim.

(24) 6. Suppose that $f(x) = \dfrac{x^2 + 3}{x^2 + x + 4}$.

(a) What is the domain of $f(x)$? Why?

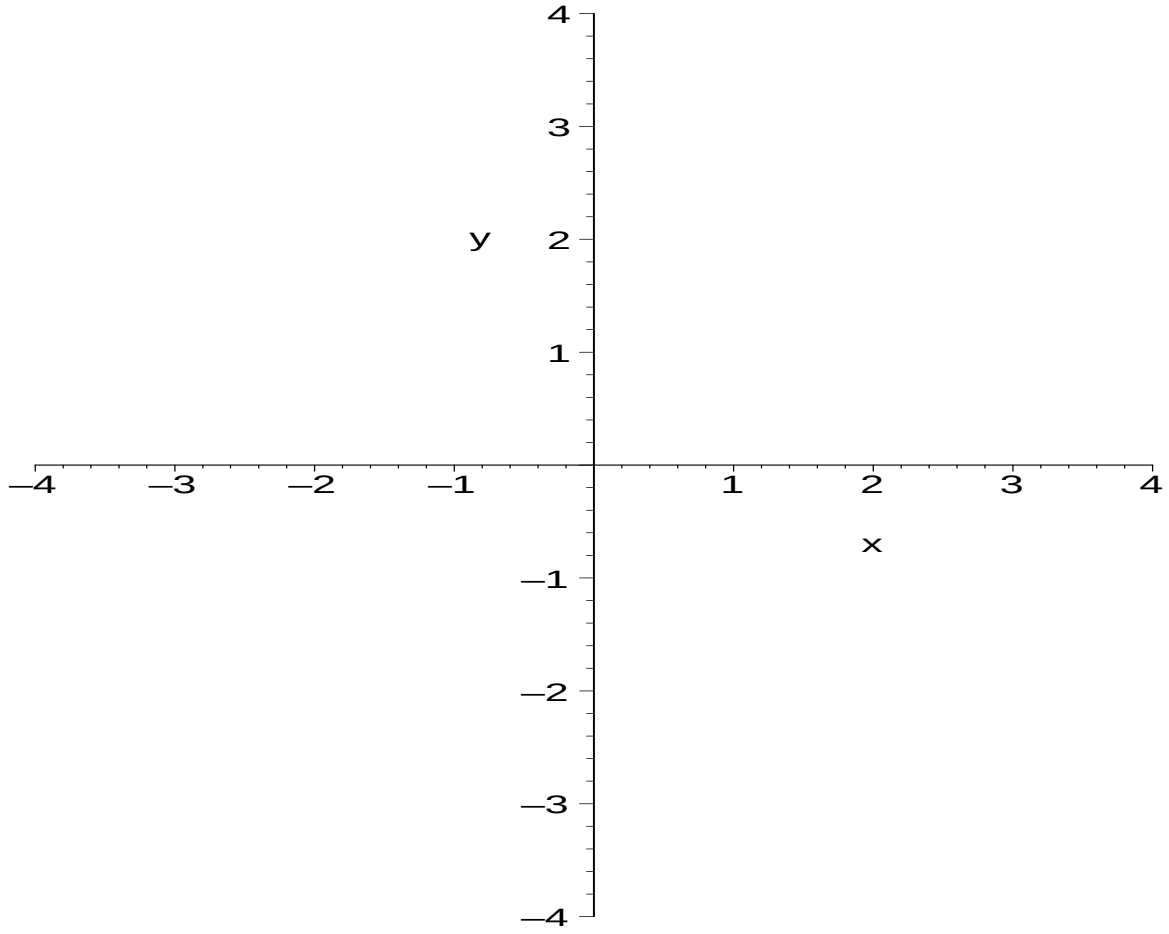
(b) What are $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$? Why?

(c) Use calculus to find all relative extreme values of $f(x)$.

(d) The *range* of a function is the collection of all possible values of that function. What is the range of f? Explain your answer carefully.

(15) 7. You wish to build a shed in the shape of a rectangular box with a square floor. The materials for the walls cost \$1 per square foot and the materials for the floor and roof cost \$2 per square foot. You want the shed to have a volume of 250 cubic feet. What should the dimensions of the shed be in order to minimize the cost of materials?

(8) 8. On the axes below sketch the graph of a function f with the following properties: The domain of f is $(-4, 4)$ and f is differentiable at all points in its domain. f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$. At $x = 0$ there is a horizontal tangent line and a point of inflection.



What is the total number of points of inflection of the function whose graph you have sketched?