(6) 1. Compute the derivatives of the following functions:

a) $xe^{\cos x}$

$$xe^{\cos x}(-\sin x) + e^{\cos x}$$

b) $\tan^3(x^3)$

$$3\tan^2(x^3)\sec^2(x^3)3x^2$$

(8) 2. Compute the following limits:

a) $\lim_{x\to 0} \frac{e^{5x} - 5x - 1}{x^2}$

This is an indeterminate form of type $\frac{0}{0}$. By L'Hôpital's Rule, used twice, this is

$$\lim_{x\to 0} \frac{5e^{5x} - 5}{2x} = \lim_{x\to 0} \frac{25e^{5x}}{2} = \frac{25}{2}$$

b) $\lim_{x\to 1} \frac{x^2 + 1}{x + 1}$

$$\frac{2}{2} = 1$$

(13) 3. Find an equation for the line tangent to the graph of $\ln y + x^3 + 2xy = 12$ at the point $(2, 1)$.

Differentiating implicitly, we get

$$\frac{y'}{y} + 3x^2 + 2xy' + 2y = 0.$$

Setting $x = 2$ and $y = 1$, we obtain

$$y' + 12 + 4y' + 4 = 0.$$

Solving for $y'$, we find that $y' = -16/5$ at the point $(2, 1)$. An equation for the tangent is $y - 1 = -\frac{16}{5}(x - 2)$.

(8) 4. A certain function $f(x)$ is defined and differentiable for all real numbers $x$. If $f(1) = 2$ and $|f'(x)| \leq 3$ for $1 < x < 3$, what is the largest possible value of $f(3)$? What is the smallest possible value of $f(3)$? Give brief explanations of your answers.

By the Mean Value Theorem, there is a number $c$ in $(1, 3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3 - 1} = \frac{f(3) - 2}{2}.$$

This means that $f(3) = 2+2f'(c)$. The biggest value $f'(c)$ can have is 3, so the biggest value $f(3)$ can have is $2+2(3) = 8$. Similarly, the smallest value $f(3)$ can have is $2+2(-3) = -4$.

(10) 5. Below is a portion of the graph of a function $f$.

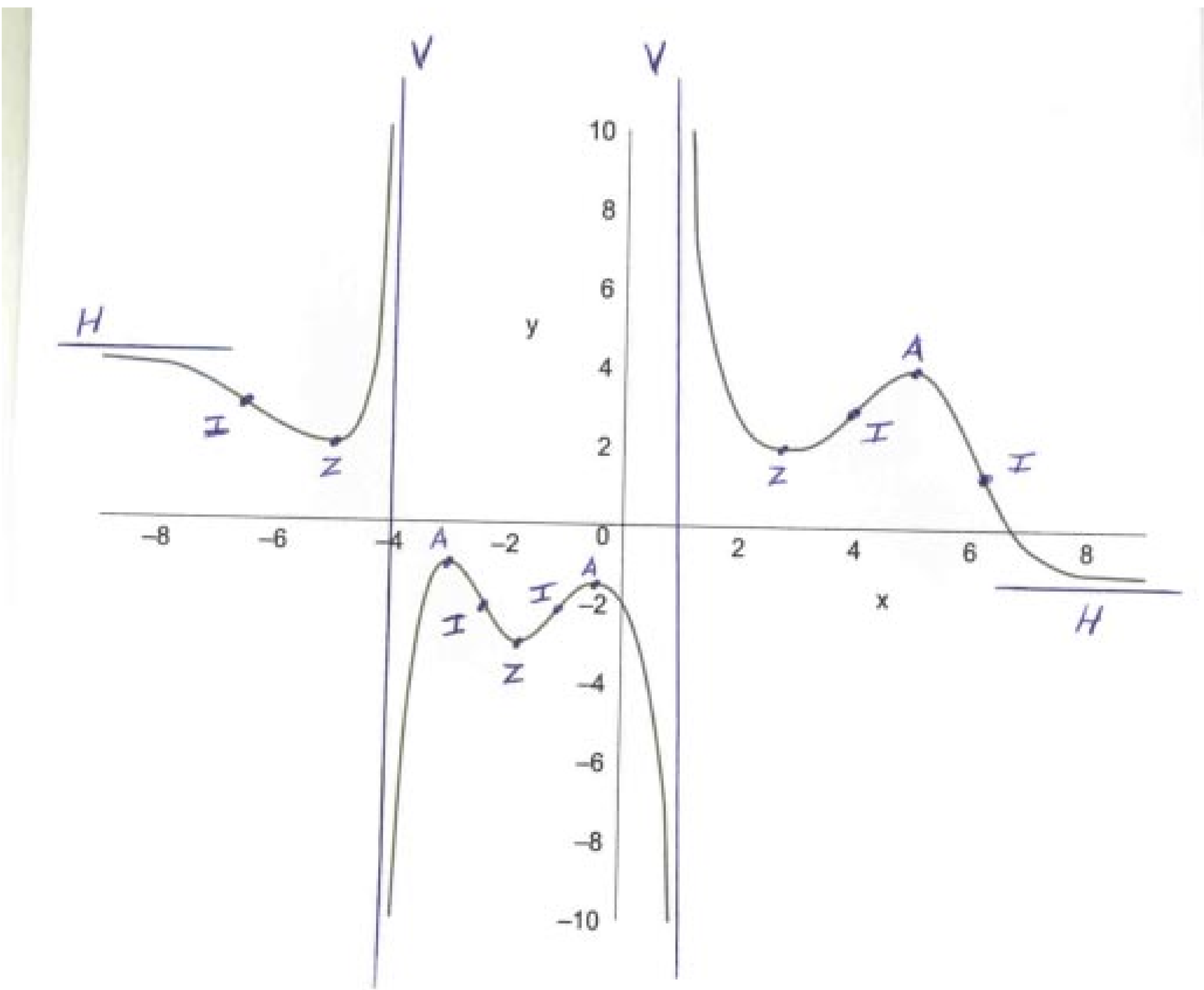
a) On the plot, draw lines that appear to be vertical asymptotes of the graph. Label each of the lines with the letter V.

b) On the plot, draw lines that appear to be horizontal asymptotes of the graph. Label each of the lines with the letter H.

c) On the graph of the function, place a small dot at each place the function has a relative maximum. Label each of these points with the letter A.

d) On the graph of the function, place a small dot at each place the function has a relative minimum. Label each of these points with the letter Z.

e) On the graph of the function, place a small dot at each place the function has a point of inflection. Label each of these points with the letter I.



(15) 6. What are the absolute maximum and the absolute minimum of the function $x^3 - 3x^2 + 7$ on the interval $[1, 4]$?

If $f(x) = x^3 - 3x^2 + 7$, then

$$f'(x) = 3x^2 - 6x = 3x(x-2).$$

Thus the critical numbers for $f$ are 0 and 2. However, of these, only 2 lies in the interval $[1, 4]$. Evaluating $f$ at 2 and the endpoints of the interval, we get

$$f(1) = 5, \qquad f(2) = 3, \qquad f(4) = 23.$$

Thus the absolute maximum value is 23 and the absolute minimum value is 3.

(10) 7. Suppose that $f(x) = e^{3x^2-3}$.

Compute $f(1)$.

$$f(1) = e^0 = 1$$

Compute $f'(1)$.

$$f'(x) = e^{3x^2-3}6x, \quad \text{so} \quad f'(1) = 6.$$

Use the linearization (differential, tangent line approximation) of $f$ at $x = 1$ to estimate $f(1.05)$.

The linearization is $L(x) = 1 + 6(x-1)$ and

$$L(1.05) = 1 + 6(0.05) = 1.30.$$

(15) 8. For some mysterious reason the dimensions of a rectangular box are changing. At a certain moment, the length is increasing at a rate of 2 feet per hour, the width is decreasing at a rate of 3 feet per hour, and the height is increasing at a rate of 4 feet per hour. If at that moment the length is 5 feet, the width is 6 feet, and the height is 3 feet, how fast is the volume of the box changing? (Be sure to give the units.) Is the volume increasing or decreasing? (Note: The volume of a rectangular box is the product of the length, the width, and the height.)

The volume $V$ is $LWH$, where $L$, $W$, and $H$ are the length, width, and length, respectively. Using the product rule twice, we have

$$V' = L'WH + L(WH)' = L'WH + L(W'H + WH') = L'WH + LW'H + LWH'.$$

At the moment described,

$$V' = (2)(6)(3) + (5)(-3)(3) + (5)(6)(4) = 101 \text{ cubic feet per hour.}$$

The volume is increasing.

(15) 9. A manufacturer can produce shoes at a cost of \$50 a pair and estimates that if the shoes are sold for $p$ dollars a pair, then consumers will buy approximately

$$1000e^{-0.1p}$$

pairs of shoes each week. At what price should the manufacturer sell the shoes to maximize profits?

If a pair is sold at a price of $p$ dollars, then the profit per pair is $p - 50$. At the price $p$, the manufacturer will sell

$$1000e^{-0.1p}$$

pairs. Thus the manufacturer's weekly profit will be

$$P = 1000e^{-0.1p}(p - 50).$$

Now

$$\frac{dP}{dp} = 1000[e^{-0.1p} + e^{-0.1p}(-0.1)(p - 50)] = 1000e^{-0.1p}[1 - \frac{1}{10}(p - 50)].$$

Hence if $\frac{dP}{dp}$ is 0, then

$$1 - \frac{1}{10}(p - 50) = 0$$

or $p$ is 60. The only realistic endpoint is $p = 50$, but at that price the manufacturer has no profit. Thus to maximize profits, the manufacturer should sell the shoes for \$60 per pair.