Sims 2006 Exam 1, no solutions

1A

(10) 1. Suppose $f(x) = 2x^2 - 3x$. Use the **definition of derivative** to find $f'(x)$.

(9) 2. Find an equation for the line tangent to the graph of $y = \sqrt{x} + 2x^2$ at the point where $x = 1$.

(12) 3. Assume that the functions $u(x)$ and $v(x)$ are defined and differentiable for all real numbers x. The following data is known about u, v, and their derivatives.

x	$u(x)$	$v(x)$	$u'(x)$	$v'(x)$
2	3	4	-1	2
3	2	1	3	-1
4	1	3	0	-2

Define $f(x) = u(x)^2 + 2v(x)$ and $g(x) = v(x)/u(x)$. Answer the following, giving a brief explanation of how the answers were obtained.

a) What is $f'(2)$?

b) What is $g'(3)$?

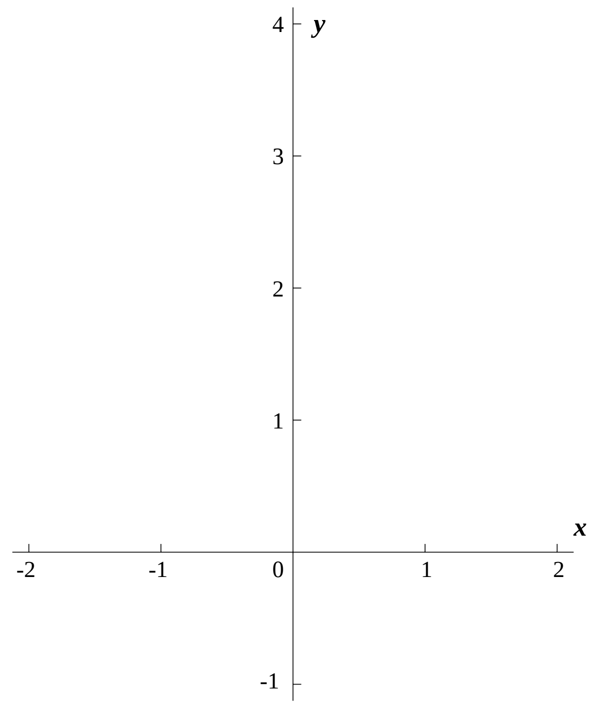
c) What can be said about the number and location of solutions to the equation $f(x) = 6.5$ with x in $[2, 4]$?

(12) 4. Suppose that the function $f(x)$ is described by

$$f(x) = \begin{cases} x + B & \text{if } x < 1 \\ Ax + 3 & \text{if } x \geq 1 \end{cases}.$$

a) Find A and B so that $f(x)$ is continuous for all numbers and $f(-1) = 0$. Briefly explain your answer.

b) Sketch $y = f(x)$ on the axes given for the values of A and B found in a) when x is in the interval $[-2, 2]$.



(16) 5. Evaluate the indicated limits exactly. Give evidence to support your answers without appealing to calculator computations, to graphing, or to l'Hôpital's Rule.

a) $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4}$

b) $\lim_{x \to 2^-} \dfrac{|x - 1| - 1}{|x - 2|}$

c) $\lim_{x \to 0} \dfrac{\sin^2 2x}{x^2}$

d) $\lim_{x \to 0} \dfrac{\cos 3x - 1}{x}$

(14) 6. In the following, distances are measured in feet and time in seconds. A particle is moving on the x-axis. Its position at time t is given by $s(t) = 2t^3 - 3t^2 - 12t + 7$.

a) What is the net distance traveled by the particle from $t = 1$ to $t = 3$?

b) What is the total distance traveled by the particle from $t = 1$ to $t = 3$?

(10) 7. Solve the following two equations for x.

a) $4^{2x-3} = 8^{x+1}$

b) $\ln(x - 2) + \ln(x + 1) = \ln(3x - 2)$

(8) 8. (There is no single correct answer to this problem.) On the axes below, sketch the graph of a function $f(x)$ with all the following properties:

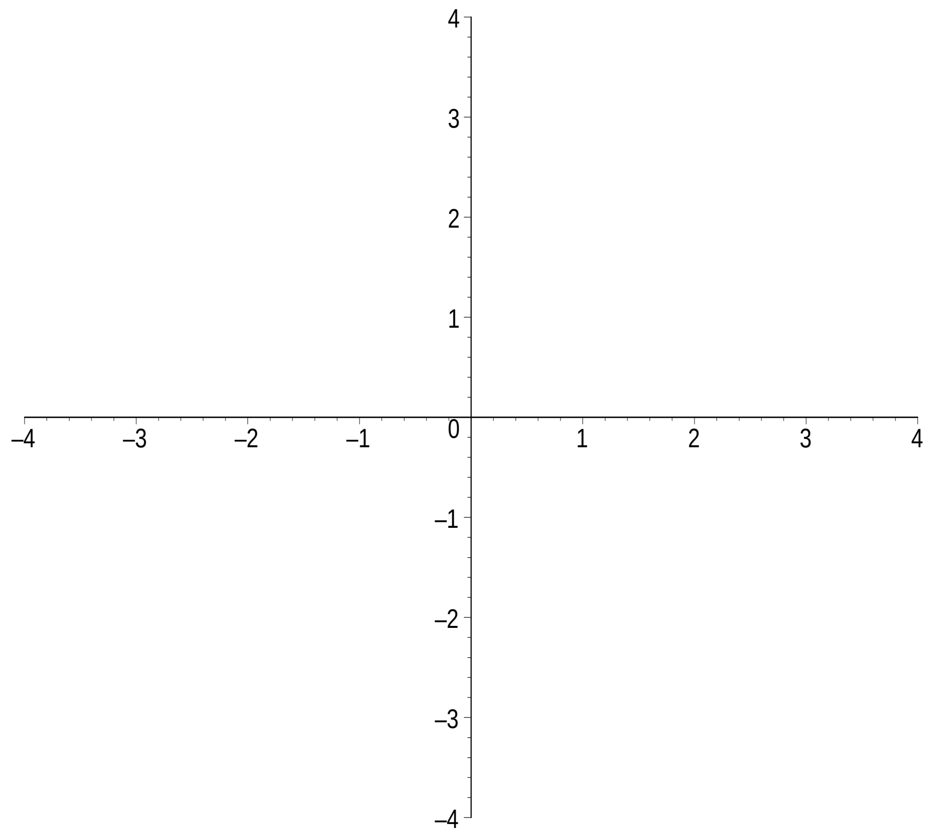
a) The domain of $f(x)$ is $[-4, 4]$.

b) $f(x)$ is differentiable at all points of its domain except $x = -1$ and $x = 2$.

c) $f(x)$ is not continuous at $x = -1$.

d) $f(x)$ is continuous but not differentiable at $x = 2$.

e) $f(0) = 1$ and $f'(0) = -1$.



(9) 9. a) If $f(x) = 2x^2\sqrt{x} + \dfrac{3}{x^3\sqrt{x}}$, what is $f'(x)$?

b) If $f(x) = \dfrac{2\tan x - 3\sec x}{\ln x}$, what is $f'(x)$?

c) If $f(x) = xe^x \sin x$, what is $f'(x)$?