Sims 2005 Exam 1, no solutions

1A

(10) 1. Suppose $f(x) = \dfrac{3}{x+2}$. Use the **definition of derivative** to find $f'(x)$.

(9) 2. Find an equation for the line tangent to the graph of $y = \dfrac{4x}{2+x^2}$ at the point where $x = 1$.

(12) 3. Assume that the functions $u(x)$ and $v(x)$ are defined and differentiable for all real numbers x. The following data is known about u, v, and their derivatives.

x	$u(x)$	$v(x)$	$u'(x)$	$v'(x)$
2	3	4	-1	2
3	2	1	3	-1
4	1	3	0	-2

Define $f(x) = u(x)v(x)$, $g(x) = u(x)/v(x)$, and $h(x) = u(v(x))$. Give the values of the following with a brief indication of how they were obtained:

a) $f'(2)$

b) $g'(3)$

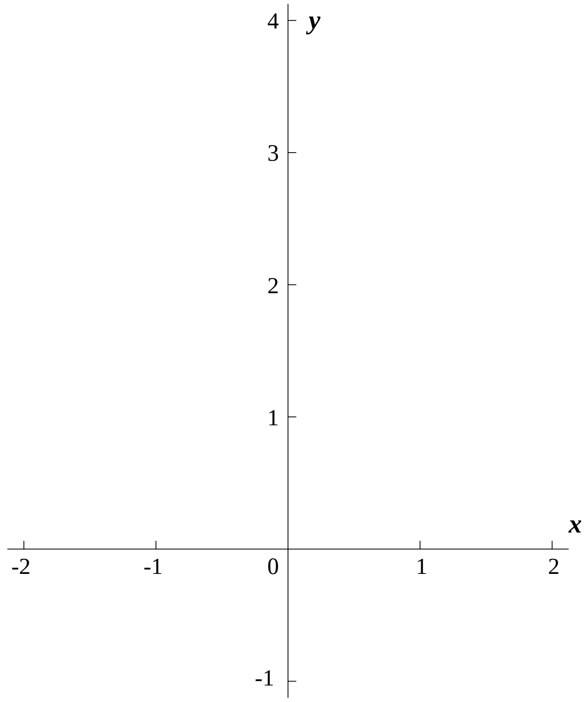
c) $h'(4)$

(14) 4. Suppose that the function $f(x)$ is described by

$$f(x) = \begin{cases} 3 - x^2 & \text{if } x < 0 \\ Ax + B & \text{if } 0 \le x \le 1 \\ 2^x & \text{if } 1 < x \end{cases}.$$

a) Find A and B so that $f(x)$ is continuous for all numbers. Briefly explain your answer.

b) Sketch $y = f(x)$ on the axes given for the values of A and B found in a) when x is in the interval $[-2, 2]$.



(20) 5. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) $\lim\limits_{x \to 1} \dfrac{x^2 + 2x - 3}{x - 1}$

b) $\lim\limits_{x \to 2^+} \dfrac{|x - 1| - 1}{|x - 2|}$

c) $\lim\limits_{x \to 0} \dfrac{\sin 2x}{\tan 3x}$

d) $\lim\limits_{x \to 4} \dfrac{3x - 2}{\cos(\pi x)}$

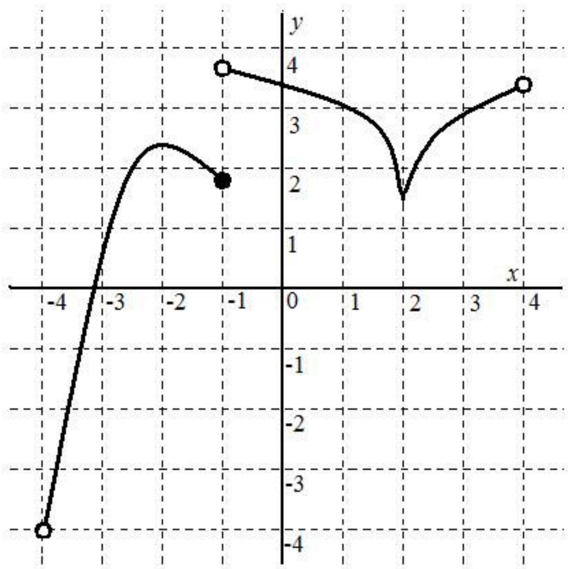
(10) 6. Suppose that $f(x)$ is defined and continuous for all real numbers x and assume that $f(x)$ takes on the following values: $f(-2) = 6$, $f(0) = -3$, $f(2) = 4$, $f(3) = 0$, $f(4) = -1$, $f(7) = -3$, and $f(10) = 8$.

a) What can be said about the number of solutions to the equation $f(x) = 0$?

b) Give a list of nonoverlapping intervals in which solutions to the equation $f(x) = 0$ can be found.

(8) 7. What is the domain of $f(x) = \dfrac{\ln x + \sqrt{4 - x}}{\sin x}$? Give your answer as a list of intervals. Explain how you arrived at your answer.

(8) 8. In this problem the function $f(x)$ has domain the open interval $(-4, 4)$. A graph of $y = f(x)$ is displayed below. Answer the following questions as well as you can based on the information in the graph.



a) For which x is $f(x)$ *not* continuous?

ANSWER: ________________

b) For which x is $f(x)$ *not* differentiable?

ANSWER: ________________

c) For which x is $f'(x) = 0$?

ANSWER: ________________

d) For which x is $f'(x) > 0$?

ANSWER: ________________

(9) 9. a) If $f(x) = \dfrac{1 - e^x}{x^2 + 1}$, what is $f'(x)$?

b) If $f(x) = (2x + 3\cos x)(x^4 - x^2)$, what is $f'(x)$?

c) If $f(x) = \sec(x^3 + 2x)$, what is $f'(x)$?