

- (13) 1. Find an equation for the tangent line to the graph of $y^2 = x^3 - 3xy + 3$ at the point $(-2, 1)$.

- (10) 2. Find equations for all vertical and horizontal asymptotes of the function

$$f(x) = \frac{3e^x + 5}{7e^x - 2}.$$

(All numbers used should be described by exact expressions, not decimal approximations. Thus you should write $\sqrt{2}$, not 1.414.)

- (15) 3. At a certain time, the length of a rectangle is 5 feet and its width is 3 feet. At that same moment, the length is *decreasing* at 0.5 feet per second and the width is *increasing* at 0.4 feet per second.

What is the length of the diagonal at that time?

How fast is the length of the diagonal changing? Is this length increasing or decreasing?

- (10) 4. Suppose that $f(x) = \sqrt{2 + 7x^3}$.

Compute $f(1)$.

Compute $f'(1)$.

Use the linearization (differential, tangent line approximation) of f at $x = 1$ to estimate $f(1.08)$.

- (5) 5. A friend runs up to you and excitedly explains that she has found a function g with the following properties:

g is continuous on $[0, 1]$ and differentiable on $(0, 1)$.

$g(0) = 1$ and $g(1) = 5$.

$g'(x) \leq 3$ for all x in $(0, 1)$.

Explain why you doubt your friend's claim.

- (24) 6. Suppose that $f(x) = \frac{x^2 + 3}{x^2 + x + 4}$.

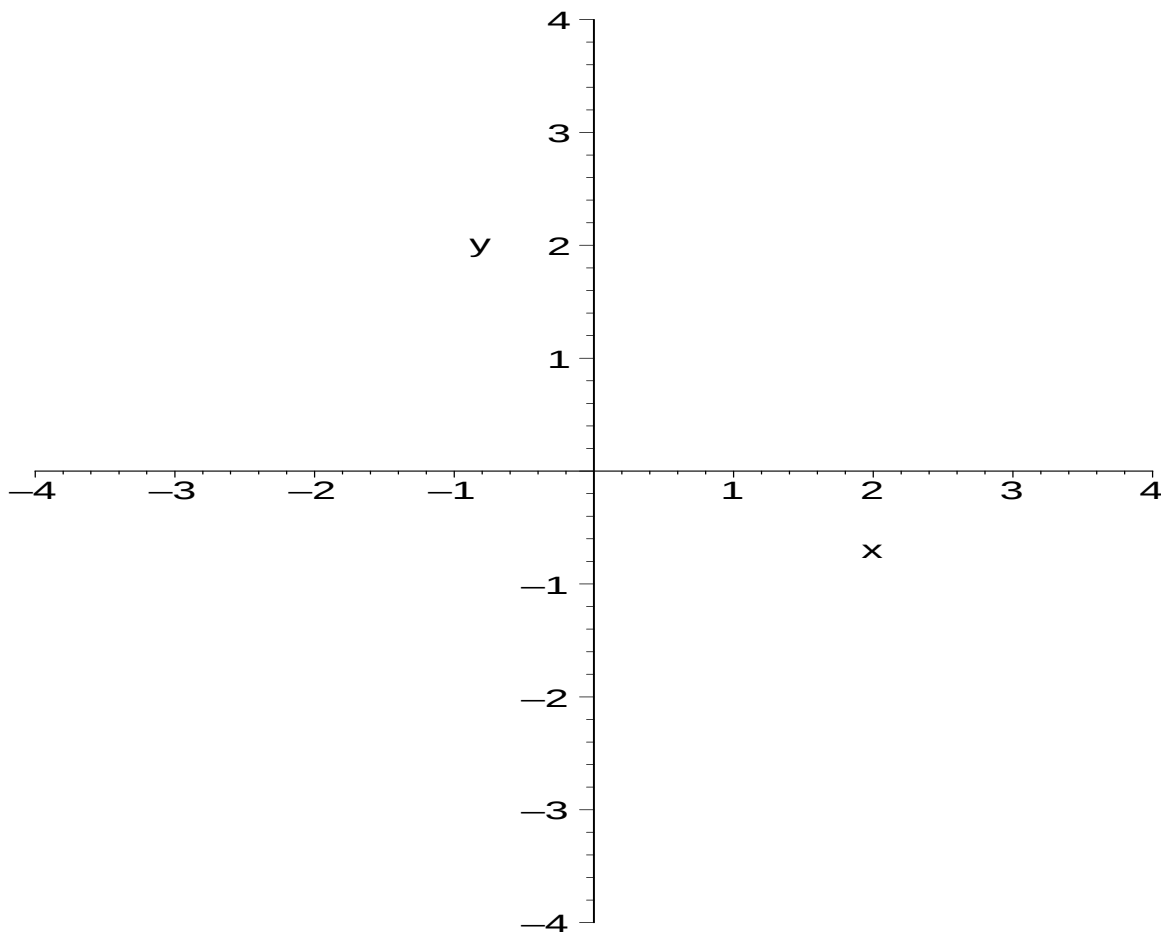
(a) What is the domain of $f(x)$? Why?

(b) What are $\lim_{x \rightarrow +\infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$? Why?

(c) Use calculus to find all relative extreme values of $f(x)$.

(d) The *range* of a function is the collection of all possible values of that function. What is the range of f ? Explain your answer carefully.

- (15) 7. You wish to build a shed in the shape of a rectangular box with a square floor. The materials for the walls cost \$1 per square foot and the materials for the floor and roof cost \$2 per square foot. You want the shed to have a volume of 250 cubic feet. What should the dimensions of the shed be in order to minimize the cost of materials?
- (8) 8. On the axes below sketch the graph of a function f with the following properties:
The domain of f is $(-4, 4)$ and f is differentiable at all points in its domain. f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$. At $x = 0$ there is a horizontal tangent line and a point of inflection.



What is the total number of points of inflection of the function whose graph you have sketched?