(15) 1. Suppose  $f(x) = x^3 - 3x^2 + 5$ . Be sure to answer all parts of this question.

a) Find all critical numbers of f(x). Describe the intervals  $20 \downarrow y$ where f(x) is increasing and where f(x) is decreasing. -1 0 1 2 4 b) Find all inflection points of f(x). Describe the intervals<sup>4</sup> -3 -2 3 where f(x) is concave up and where f(x) is concave down. -20 c) Sketch a graph of y = f(x) on the axes given. Label -40 any relative maxima with an M on your graph. Label -60 any relative minima with an **m** on your graph. Label any -80 inflection points with an I on your graph. Note The units on the horizontal and vertical axes are -100 different.

(12) 2. Find the equations of all vertical and horizontal asymptotes of  $f(x) = \frac{3e^x + 5}{7e^x - 2}$ .

Numerical approximations are *not* acceptable.

- (10) 3. At a certain time, the length of a rectangle is 3 feet and its width is 5 feet. At that time, the length is *increasing* at .4 feet per second, and the width is *decreasing* at .5 feet per second. What is the length of the diagonal of the rectangle at that time? At that time, how fast is the length of the rectangle's diagonal changing? Is this length increasing or decreasing?
- (12) 4. A graph of a portion of the curve described implicitly by the equation  $y^2 + x^3 + y + y \sin(x + x^2) = 2$  is shown to the right.

a) Find the coordinates of the two points of intersection of this curve with the y-axis (where x = 0).

b) Are the tangent lines to  $y^2 + x^3 + y + y \sin(x + x^2) = 2$  at the two points where the curve intersects the y-axis parallel? Use calculus to answer this question.

- (8) 5. Suppose  $f(x) = \sqrt{2 + 7x^3}$ .
  - a) Compute f(1).
  - b) Compute f'(1).

c) Find an approximate value for f(1.08) using the differential or tangent line approximation for f(x) and your answers to a) and b).





- (8) 6. Suppose g(x) is a differentiable function, and we know that g(1) = 2, g'(1) = 3, and g''(1) = -1. Define the function h(x) by the equation  $h(x) = g(2x^3 x^2)$ . Compute h(1), h'(1), and h''(1).
- (18) 7. Suppose  $f(x) = \frac{x^2 + 3}{x^2 + x + 4}$ .
  - a) What is the domain of f(x)? Why?
  - b) What is  $\lim_{x \to +\infty} f(x)$ ? Why? What is  $\lim_{x \to -\infty} f(x)$ ? Why?
  - c) Use calculus to find any relative maximum and relative minimum values of f(x).

d) The range of a function is the collection of all possible values of the function. What is the exact range of f(x)? Explain your answer using calculus.

(9) 8. Suppose the sum of two non-negative numbers is 15. Verify using calculus that the largest the product of the square of one multiplied by the other can be is 500.

(8) 9. Suppose that 
$$F(x) = (2x^3 + 8)^{12} - (7 - 3x^5)^{15}$$
.

a) Compute F'(x).

b) Use your answer to a) and calculus to explain briefly why F(200) is less than F(400).

## Second Exam for Math 135, sections 21, 22, and 23

April 8, 2005

NAME \_\_\_\_\_

SECTION (please circle one) 21 22 23

Do all problems, in any order.

Show your work. An answer alone may not receive full credit. No notes other than the distributed formula sheet may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	15	
2	12	
3	10	
4	12	
5	8	
6	8	
7	18	
8	9	
9	8	
Total Points Earned:		