Name ______________________________ MA135 Final Exam Dec. 17, 2007

1. Find the following limits (5 points each), giving reasons for your answers. You may use any method from this course.

a. $\lim_{h \to 0} \dfrac{\sqrt{5+h} - \sqrt{5}}{h} =$ ______________

b. $\lim_{x \to 1} \dfrac{\frac{1}{x} - 1}{\sqrt{x} - 1} =$ ______________

c. $\lim_{x \to \infty} \dfrac{x^2 + \sin(x)}{x^2 + 1} =$ ______________

2. Find the derivatives of the following functions (5 points each). You do not need to simplify your answers.

a. If $y = \sqrt{1 + x^2}$ then $\dfrac{dy}{dx} =$

b. If $y = \dfrac{e^x + x}{e^x + x^2}$ then $\dfrac{dy}{dx} =$

c. If $y = x^{\cos(x)}$ then $\dfrac{dy}{dx} =$

3. Find the following indefinite integrals (5 points each).

a. $\int x(1 + \sqrt{x})\, dx =$ ______________

b. $\int (3x^2 + 1) \sin(x^3 + x)\, dx =$ ______________

c. $\int \dfrac{\sin(x)\, dx}{\cos(x) + 2} =$ ______________

4. Calculate the following definite integrals (5 points each).

a. $\int_1^2 \dfrac{1 + \sqrt{x}}{x}\, dx =$ ______________

b. $\int_0^{\sqrt{\pi/2}} x \sin(x^2)\, dx =$ ______________

c. $\int_2^3 \dfrac{(\ln(x))^3}{x}\, dx =$ ______________

5. (14 points) Compute the value of the Riemann sum for the function $f(x) = x^2$ on the interval $[1, 3]$ using $n = 4$ and taking x_k^* to be the midpoint of the k^{th} interval in the partition. You can leave your answer as a sum of fractions.

6. (7 points each) Find the equations of the lines tangent and perpendicular to the curve described by

$$x^2y^3 + 3 = 5y^2 + x$$

at the point $(2, 1)$. Any form of the equation will do.

a. The equation of the tangent line is ___________________

b. The equation of the perpendicular line is ___________________

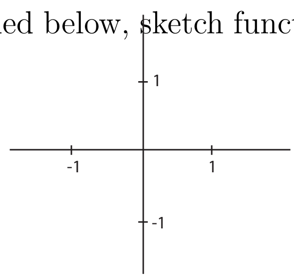
7. (7 points each) Please give reasons for your answers.

a. $\lim\limits_{x \to 0^+} (e^x + x)^{1/x} =$ ___________________

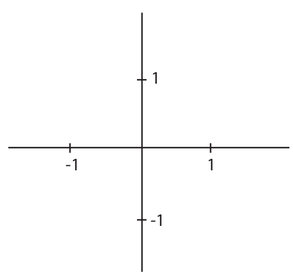
b. $\lim\limits_{x \to \frac{\pi}{2}^-} \left(x - \dfrac{\pi}{2}\right) \tan x =$ ___________________

8. (4 points each) On the axes provided below, sketch functions having the required properties

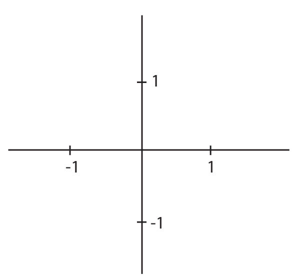
a. $f(0) = 1,\ f'(0) > 0,\ f''(0) > 0$



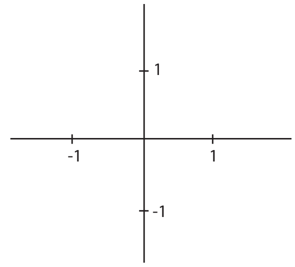
b. $f(0) = 1,\ f'(0) < 0,\ f''(0) < 0$



c. $f(0) = 1,\ f'(0) = 0,\ f''(0) < 0$



d. $f(0) = 1$, $f'(0) > 0$, $f''(0) = 0$, $f''(x) > 0$ for $x < 0$
$f''(x) < 0$ for $x > 0$



9. (14 pts) In a healthy person of height x in, the average pulse rate in beats per minute is modeled by the formula

$$P(x) = \frac{596}{\sqrt{x}} \qquad 30 \leq x \leq 100$$

Use linear approximation to estimate the change in pulse that corresponds to a height change from 59 to 60 in.

10. (14 pts) A viticulturist estimates that if 50 grapevines are planted per acre, each grapevine will produce 140 lb of grapes. Each additional grapevine planted per acre (up to 20 extra vines) reduces the average yield per vine by 2 lb. How many grapevines should be planted to maximize the yield per acre?

11. (7 points each) a. The marginal cost of a certain commodity is

$$C'(x) = 3x^2 - 36x + 120.$$

If it costs \$5 to produce 1 unit, what is the total cost of producing x units?

$C(x) =$ __________________

b. For what value of x is the marginal cost a minimum? $x =$ ____________ Explain briefly why your answer is a minimum.

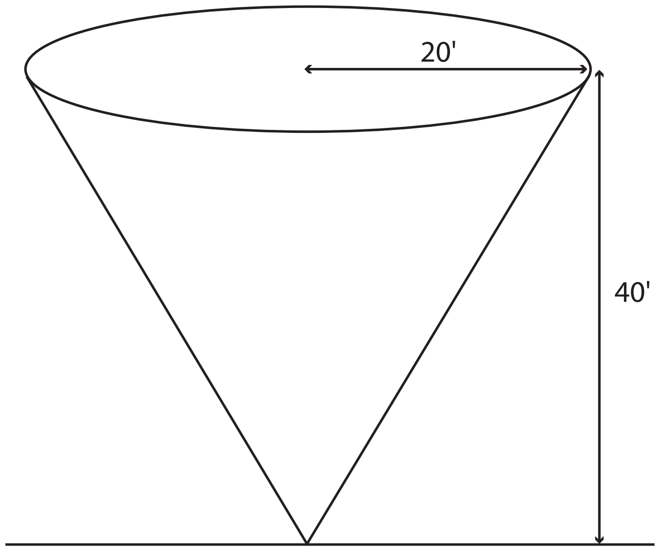
12. (15 points) Suppose that U is a differentiable function with $U(9) = 5$ and $U'(9) = 3$ and that $V(x) = U(x^2)$.

a. (5 pts) What is $V(3)$? $V(3) =$ ____________

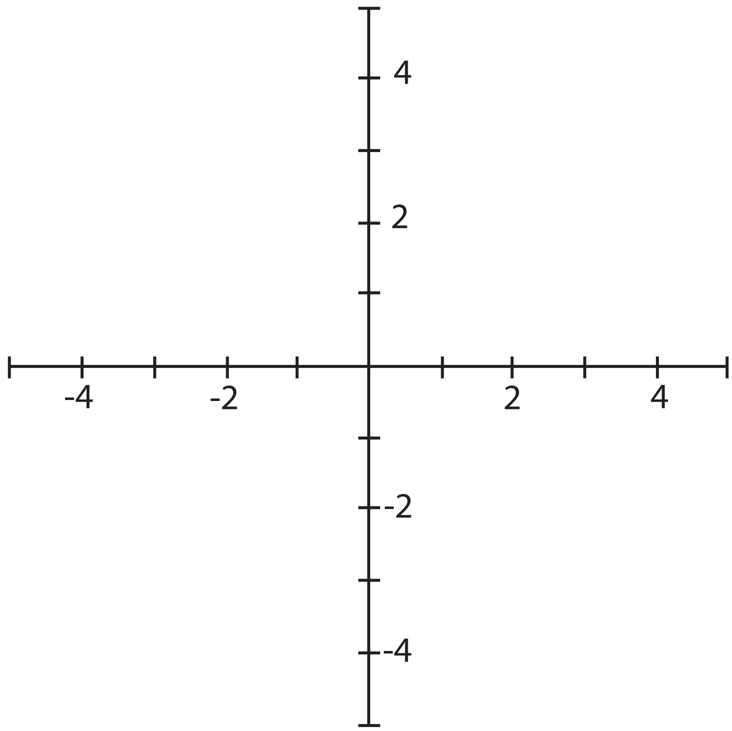
b. (5 pts) What is $V'(3)$? $V'(3) =$ ____________

c. (5 pts) If we know that $V''(3) = 7$, what is $U''(9)$? $U''(9) =$ ____________

13. (14 points) A conical water tank is 40 feet high and has a radius of 20 feet at the top. If water flows into the tank at 80 cubic feet per minute, how fast is the water rising when the water in the tank is 12 feet deep?



14. (15 points) Plot the function $f(x) = \dfrac{x^2}{4 - x^2}$ on the axes below. Use as many as possible of the the key features such as domain, regions where $f(x)$ is increasing and decreasing, relative extrema, inflection points, concavity, asymptotes, and intercepts.



a. domain = ______________

b. inc = ______________

c. dec = ______________

d. asymptotes = ______________

e. relative extrema = ______________

f. inflections = ______________

g. concave up = ______________

h. concave down = ______________

i. intercepts = ______________