Practice problems for the second exam in Math 135, Fall 2015

In addition to these problems, it is imposrtant for the students to study assigned homework, webwork problems, examples in the book, review problems at the end of each chapter, and Professor Sim's and Professor Greenfield's sample exams.

1. a) Let $y = f(x)$ be defined by the equation $e^{xy} + \ln y^2 = x$. Find $\dfrac{dy}{dx}$.

   b) Find the equation of the tangent line to the graph of $x^3 + y^3 = y + 21$ at $(3, -2)$.

   c) If $y = x^{(x^2)}$, Find $\dfrac{dy}{dx}$.

2. a) One end of a rope is fastened to a boat and the other end is wound around a windlass located on a dock at a point 4 meter above the level of the boat. If the boat is drifting away from the dock at the rate of 2 m/min, how fast is the rope unwinding at the instant when the length of the rope is 5 meter?

   b) A ladder of 5 meter long rests against a vertical wall. If the foot of the ladder moves away from the wall at a rate of 2 m/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 4 meter from the wall?

3. a) Use differential to approximate $\sqrt{103}$.

   b) Find $d(e^{4x}\sin(x^3))$.

4. a) Find the absolute maximum and minimum of $f(x) = 8x^5 - 5x^4$ on $[-1, 1]$.

   b) Find the absolute maximum and minimum of

$$f(x) = \begin{cases} 9 - 4x & \text{if } 0 \le x < 1 \\ -x^2 + 6x & \text{if } 1 \le x \le 4 \end{cases}$$

   on $[0, 4]$.

5. Let $f(x) = \dfrac{1}{\sqrt{x}}$. Find the number c in the Mean Value Theorem if $a = 1$ and $b = 4$.

6. For the following functions, find all the relative maxima, relative minima, inflection points, horizontal asymptotes and vertical asymptotes:

   a) $f(x) = e^{-x^2}$.

   b) $f(x) = \dfrac{x^2 - x - 2}{x - 3}$.

7. a) Sketch the graph of $g(x) = x^4 - 4x^3 + 10$.

   b) Sketch the graph of $f(x) = \dfrac{3x - 5}{x - 2}$.

8. Evaluate the following limits:

   a) $\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{3x}$.

   b) $\lim_{x\to 0^+}\left(\frac{2\cos x}{\sin 2x}-\frac{1}{x}\right)$.

   c) $\lim_{x\to 0}\frac{1-\cos x}{\sec x}$.

   d) $\lim_{x\to 0}\frac{x-\sin x}{x^3}$.

9. A fence must be built to enclose a rectangular area of 1,600 feet. The fence around three sides is to be made of the material that costs \$120/ft. The material for the fourth side costs \$40/ft. Find the dimensions of the rectangular that would be the least expensive to built.