Practice problems for the first exam in Math 135, Fall 2015

In addition to these problems, it is imposrtant for the students to study assigned homework, webwork problems, examples in the book, review problems at the end of each chapter, and Professor Sim's sample exams.

1. Find the following limits:

   a) $\lim\limits_{x\to 9} \dfrac{(\sqrt{x+7}-4)(x-1)}{x-9}$.

   b) $\lim\limits_{x\to 0} \dfrac{(x^2+1)\tan 2x}{\sin 4x}$.

2. Let

$$f(x) = \begin{cases} 2(x+1) & x < 3, \\ 4 & x = 3, \\ x^2 - 1 & x > 3. \end{cases}$$

   a) Find $\lim_{x\to 3} f(x)$.

   b) Is $f(x)$ continuous at $x = 3$?

3. Prove that the equation $\cos \pi x - x \sin \pi x = 2x - 1$ has at least one solution in the interval $(0, 1)$.

4. a) Find all values of x such that $4^x + 16 = 10 \cdot 2^x$.

   b) Evaluate $2^{2\log_2 3 - 3\log_2 2}$. Express the answer as a rational number.

   c) An account in a bank pays 2% interest compounded continuously. How long will it take for \$2500 to double? Express your answer using $\ln 2$ and integers.

5. a) Find the equation of the tangent and normal lines to the graph of the function $f(x) = 2x^3 - x + 1$ at $x = 1$. (The normal line is the straight line perpendicular to the tangent.)

   b) At which points on the graph is the tangent line horizontal?

6. Find the following derivatives:

   a) $\dfrac{d}{d\theta}\left(\dfrac{\theta - 1}{2 + \cos\theta}\right)$.

   b) $\dfrac{d^2}{dt^2}(t^3 e^{2t})$.

   c) $\dfrac{d}{dx}\cos(x + e^{2x})$.

7. Find a, b, c so that the function $f(x) = ax^2 + bx + c$ has x-intercepts at $(0, 0)$ and $(5, 0)$ and the slope of its tangent line at $x = 2$ is 1 .

8. The electric charge $Q(t)$ stored on a capacitor decays exponentially, that is, the electric charge after t weeks is equal to $Q(t) = Ae^{-kt}$. Assume that at $t = 0$, the electric charge stored is 4000 and the electric charge stored after 5 weeks is equal to 2000.

   a) Find A and k. Express the answer in terms of $\ln 2$ and rational numbers.

   b) What is the electric charge stored after 10 weeks? Express the answer as an integer.

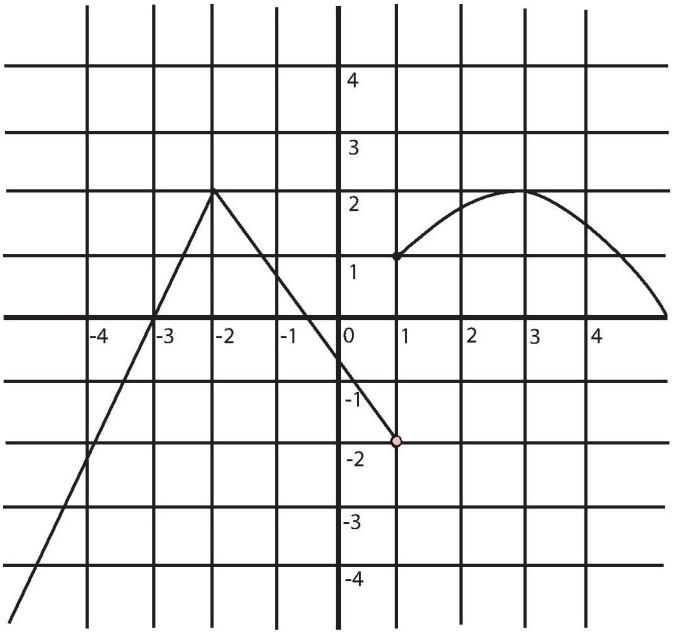
9. Suppose a person standing on the top of a building of 160 ft high throws a ball directly upward with an initial speed of 48 ft/s.

   a) Find the ball's height, velocity and acceleration at time t.

   b) When does the ball hit the ground and what is its impact velocity?

   c) How far does the ball travel during its flight?

10. The graph of a function $f(x)$ is given below:



    Find all values of x where f fails to be

    a) continuous,

    b) differentiable.

    c) For which values of x is the derivative of f equal to 0?