Name ________________________________ MA135 Final Exam A December 16, 2010

Instructor ____________________________ Section ______

Be sure to show all of your work. All solutions should use calculus techniques from this course. Unsupported answers will receive no credit! Calculators are not allowed on this exam. You may only use the formula sheet and scratch paper supplied with this exam. Good Luck!!

Prob No.	**Max Pts**	**Points**	**Prob No.**	**Max Pts**	**Points**
1	18		8	17	
2	18		9	18	
3	18		10	18	
4	18		11	18	
5	17		12	18	
6	18		13	18	
7	18		14	18	
Subtotal	125		**Subtotal**	125	

Grand Total	

1. (6 points each) Find the following limits, giving reasons for your answers. You may use any method from this course.

a. $\lim_{x \to 2} \frac{x-2}{x^2+x-6} =$ ____________________

b. $\lim_{x \to 0} \frac{\sin 3x - 3x}{x^3} =$ ____________________

c. $\lim_{x \to +\infty} \frac{e^{-x}+2}{e^{-x}+7} =$ ____________________

2. (9 points each) Find the derivatives of the following functions. You do not need to simplify your answers.

a. If $y = \cos(3x^2 + 1)$ then $\frac{dy}{dx} =$ ____________________

b. If $y = \frac{3x+2}{4x-1}$, then $\frac{dy}{dx} =$ ____________________

3. (9 points each) Find the following indefinite integrals.

a. $\int x(1+\sqrt{x})\,dx =$ ____________________

b. $\int \frac{\cos(2x)}{\sin^3(2x)}\,dx =$ ________________________________

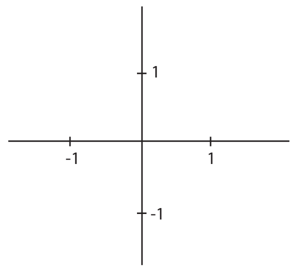
4. (9 points each).

a. $\displaystyle\int_0^1 \frac{x\,dx}{\sqrt{4-x^2}} =$ ________________________________

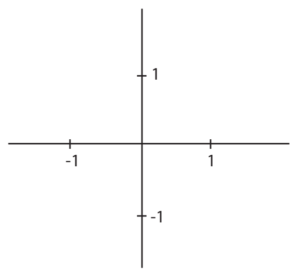
b. $\displaystyle\int_1^{e^4} \frac{dx}{x\sqrt{\ln(x)}} =$ ____________________

5. (4 points each + 1 for free) On the axes provided below, sketch functions having the required properties

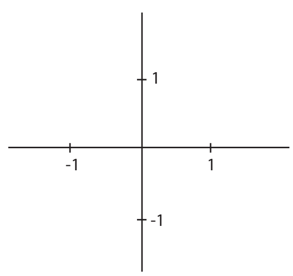
a. $f(0) = 1,\ f'(0) > 0,\ f''(0) > 0$



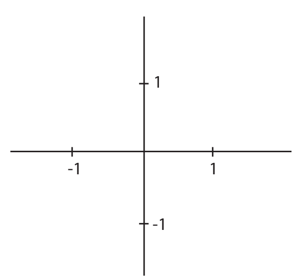
b. $f(0) = 1,\ f'(0) < 0,\ f''(0) > 0$



c. $f(0) = 1,\ f'(0) = 0,\ f''(0) < 0$



d. $f(0) = 1,\ f'(0) > 0,\ f''(0) = 0,\ f''(x) < 0$ for $x < 0$
$f''(x) > 0$ for $x > 0$



6. (18 points) Find the equations of the tangent and normal lines to the curve described by

$$x^3 + x + x^2y - 2\sin(xy) = 2$$

at the point $(1, 0)$. Any correct equation specifying this line is acceptable.

Tangent line:	
Normal line:	

7. (18 points) Find the linear approximation to $f(x) = \ln(2x)$ at $x = 1/2$. Use this approximation to estimate $f(.56)$.

8. (17 pts) Find the absolute maximum and minimum of the function $f(x) = x^4 - 8x^2 + 10$ on the interval $[-1,\ 3]$.

Absolute max:	
Absolute min:	

9. (18 points) Compute the value of the Riemann sum for the function $f(x) = x^2 + 3x$ on the interval $[1,\ 3]$ using $n = 3$ and taking x_k^* to be the left endpoint of the k^{th} interval in the partition. You can leave your answer as a sum of fractions.

Value:	

10. (6 points each)

a. Find $\dfrac{dy}{dx}$ if $y = (2x)^x$.

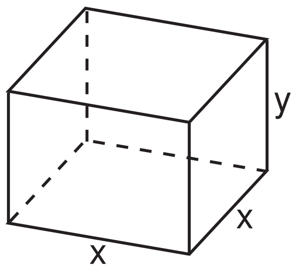
b. Find $\dfrac{dy}{dx}$ if $y = \displaystyle\int_0^x e^{t^2}\,dt$.

c. Find $\dfrac{dy}{dx}$ if $y = \displaystyle\int_0^{x^2+1} e^{t^2}\,dt$.

11. (18 points) The owner of a large kennel knows from experience that 80 incredibly cute dachshund puppies will be sold monthly if the price is 420 dollars per dog. A market survey suggests that, on average, one additional dachshund will be sold monthly for each 7 dollar decrease in price. Similarly, one less will be sold for each 7 dollar increase in price. If it costs $28 to bring each puppy to market, what price should the owner charge to maximize profit?

Price:	

12. (18 points) If 1200 square centimeters of material is available to make a box with a square base and an open top, find the largest possible volume of the box. (Hint: The surface area should be 1200 square centimeters.)



13. (18 points) At noon, ship A is 40 miles due west of ship B. Ship A is sailing west at 4 mph and ship B is sailing north at 3 mph. How fast is the distance between the ships changing at 5 PM? You do not need to simplify your answer.

14. (18 points) Consider the function $f(x) = \dfrac{x^3}{2(x^2-16)}$. For this function,

$$f'(x) = \frac{x^2(x^2-48)}{2(x^2-16)^2} \text{ and } f''(x) = \frac{16x(x^2+48)}{(x^2-16)^3}. \text{ Hint: } \sqrt{48} \simeq 6.9.$$

Horizontal asymptote(s):	
Vertical asymptote(s):	
Increasing:	
Decreasing:	
Concave up:	
Concave down:	
Relative max/min:	
Inflections:	

You only need to find the x-values for the points above.

Make a rough sketch of the graph of $y = f(x)$.