(10) 1. Suppose $f(x) = \frac{3}{x+2}$. Use the **definition of derivative** to find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{3}{x+h+2} - \frac{3}{x+2}}{h}$$

$$= \lim_{h\to 0} \frac{3(x+2) - 3(x+h+2)}{h(x+h+2)(x+2)} = \lim_{h\to 0} \frac{-3h}{h(x+h+2)(x+2)}$$

$$= \lim_{h\to 0} \frac{-3}{(x+h+2)(x+2)} = \frac{-3}{(x+2)^2}.$$

(9) 2. Find an equation for the line tangent to the graph of $y = \frac{4x}{2+x^2}$ at the point where $x = 1$.

When $x = 1$, the value of y is $4/3$.

$$\frac{dy}{dx} = \frac{(2+x^2)4 - (4x)(2x)}{(2+x^2)^2} = \frac{8-4x^2}{(2+x^2)^2}.$$

When $x = 1$, this is $4/9$. Thus an equation for the tangent is

$$y - \frac{4}{3} = \frac{4}{9}(x-1).$$

(12) 3. Assume that the functions $u(x)$ and $v(x)$ are defined and differentiable for all real numbers x. The following data is known about u, v, and their derivatives.

x	$u(x)$	$v(x)$	$u'(x)$	$v'(x)$
2	3	4	−1	2
3	2	1	3	−1
4	1	3	0	−2

Define $f(x) = u(x)v(x)$, $g(x) = u(x)/v(x)$, and $h(x) = u(v(x))$. Give the values of the following with a brief indication of how they were obtained:

a) $f'(2)$

$$f'(2) = u(2)v'(2) + u'(2)v(2) = 3\cdot 2 + (-1)\cdot 4 = 2.$$

b) $g'(3)$

$$g'(3) = \frac{v(3)u'(3) - u(3)v'(3)}{v(3)^2} = \frac{1\cdot 3 - 2\cdot(-1)}{1^2} = 5.$$

c) $h'(4)$

$$h'(4) = u'(v(4))v'(4) = u'(3)v'(4) = 3 \cdot (-2) = -6.$$

(14) 4. Suppose that the function $f(x)$ is described by

$$f(x) = \begin{cases} 3 - x^2 & \text{if } x < 0 \\ Ax + B & \text{if } 0 \le x \le 1 \\ 2^x & \text{if } 1 < x \end{cases}.$$

a) Find A and B so that $f(x)$ is continuous for all numbers. Briefly explain your answer.

The value of $f(0)$ is B and

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 3 - x^2 = 3.$$

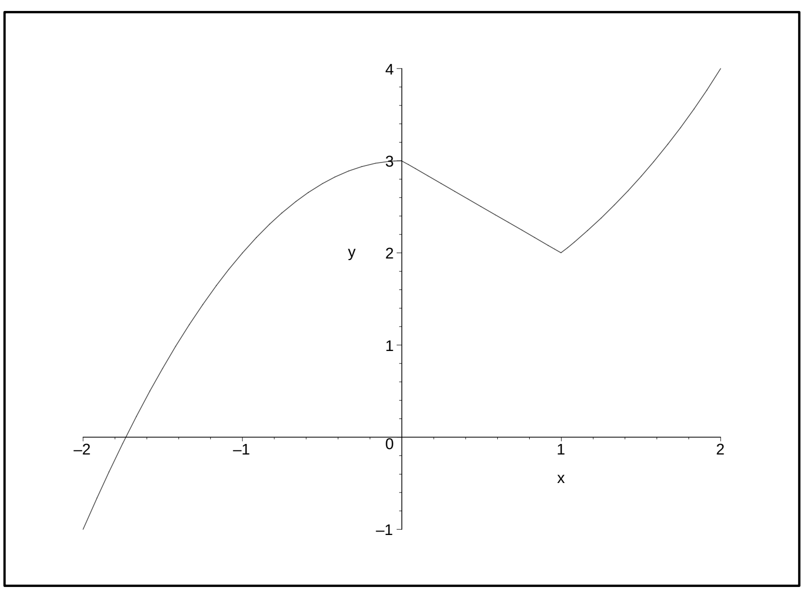
If $f(x)$ is continuous at 0, then $B = 3$.

The value of $f(1)$ is $A + B = A + 3$ and

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2^x = 2.$$

Therefore $A + 3 = 2$ or $A = -1$.

b) Sketch $y = f(x)$ on the axes given for the values of A and B found in a) when x is in the interval $[-2, 2]$.



(20) 5. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) $\lim\limits_{x\to 1} \dfrac{x^2+2x-3}{x-1}$

$$\lim_{x\to 1} \frac{x^2+2x-3}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+3)}{x-1} = \lim_{x\to 1} x+3 = 4.$$

b) $\lim\limits_{x\to 2^+} \dfrac{|x-1|-1}{|x-2|}$

If $x > 2$, then both $x-1$ and $x-2$ are positive and $|x-1| = x-1$ and $|x-2| = x-2$. Therefore

$$\lim_{x\to 2^+} \frac{|x-1|-1}{|x-2|} = \lim_{x\to 2^+} \frac{x-2}{x-2} = \lim_{x\to 2^+} 1 = 1.$$

c) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{\tan 3x}$

$$\lim_{x\to 0} \frac{\sin 2x}{\tan 3x} = \lim_{x\to 0} \frac{\sin 2x}{\frac{\sin 3x}{\cos 3x}} = \lim_{x\to 0} \frac{\sin 2x \cos 3x}{\sin 3x} = \lim_{x\to 0} \frac{2\frac{\sin 2x}{2x}\cos 3x}{3\frac{\sin 3x}{3x}} = \frac{2}{3}.$$

d) $\lim\limits_{x\to 4} \dfrac{3x-2}{\cos(\pi x)}$

$$\lim_{x\to 4} \frac{3x-2}{\cos(\pi x)} = \frac{3\cdot 4-2}{\cos(4\pi)} = 10.$$

(10) 6. Suppose that $f(x)$ is defined and continuous for all real numbers x and assume that $f(x)$ takes on the following values: $f(-2) = 6$, $f(0) = -3$, $f(2) = 4$, $f(3) = 0$, $f(4) = -1$, $f(7) = -3$, and $f(10) = 8$.

a) What can be said about the number of solutions to the equation $f(x) = 0$?

There are at least four solutions to the equation $f(x) = 0$.

b) Give a list of nonoverlapping intervals in which solutions to the equation $f(x) = 0$ can be found.

By the Intermediate Value Theorem there is at least one solution of the equation $f(x) = 0$ in each of the intervals $(-2, 0)$, $(0, 2)$, $[3, 3]$, and $(7, 10)$.

(8) 7. What is the domain of $f(x) = \dfrac{\ln x + \sqrt{4-x}}{\sin x}$? Give your answer as a list of intervals. Explain how you arrived at your answer.

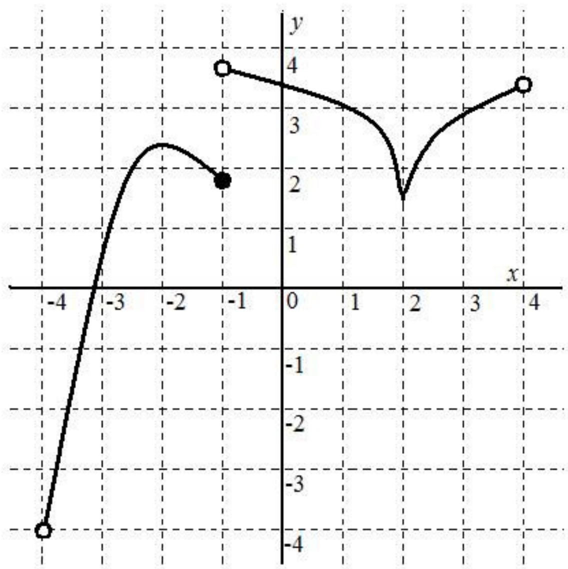
$\ln x$ is defined only for $x > 0$.

$\sqrt{4-x}$ is defined only for $x \leq 4$.

$\dfrac{1}{\sin x}$ is defined only when x is not of the form $n\pi$ for some integer n.

The numerator of $f(x)$ is defined for x in the interval $(0, 4]$. However, that interval contains one integer multiple of π, namely π itself. Thus the domain of f consists of the two intervals $(0, \pi)$ and $(\pi, 4]$.

(8) 8. In this problem the function $f(x)$ has domain the open interval $(-4, 4)$. A graph of $y = f(x)$ is displayed below. Answer the following questions as well as you can based on the information in the graph.



a) For which x is $f(x)$ *not* continuous?

$x = -1$

b) For which x is $f(x)$ *not* differentiable?

$x = -1$ and $x = 2$

c) For which x is $f'(x) = 0$?

$x = -2$

d) For which x is $f'(x) > 0$?

For those x with $0 < x < -2$ or $2 < x < 4$

(9) 9. a) If $f(x) = \dfrac{1 - e^x}{x^2 + 1}$, what is $f'(x)$?

$$f'(x) = \frac{(x^2 + 1)(-e^x) - (1 - e^x)(2x)}{(x^2 + 1)^2}.$$

b) If $f(x) = (2x + 3\cos x)(x^4 - x^2)$, what is $f'(x)$?

$$f'(x) = (2x + 3\cos x)(4x^3 - 2x) + (2 - 3\sin x)(x^4 - x^2).$$

c) If $f(x) = \sec(x^3 + 2x)$, what is $f'(x)$?

$$f'(x) = \sec(x^3 + 2x)\tan(x^3 + 2x)\,(3x^2 + 2).$$