(20) 1. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) $\lim\limits_{x\to 3} \dfrac{x^2-x-6}{x-3}$

b) $\lim\limits_{x\to -2^-} \dfrac{x^2-x-6}{|x+2|}$

c) $\lim\limits_{x\to \infty} \dfrac{4x-7}{e^{3x}}$

d) $\lim\limits_{x\to 0} \dfrac{\sin(3x)}{\cos(2x)}$

(12) 2. Suppose $f(x)=\frac{1}{x^2}$. Use the **definition of derivative** to find $f'(x)$.

(22) 3. Compute the derivatives with respect to x of the following functions.
Please do not simplify the answers.

a) $\cos\left(x^4+3\right)$

b) $\left(e^{7x}+3x^4\right)\left(\sqrt{x+5}+\dfrac{2}{x^3}\right)$

c) $\dfrac{x^3+2}{5\ln x}$

d) $\displaystyle\int_{-3}^{x} \sin\left(t^3\right)\,dt$

(8) 4. Suppose $f(x)=\left(x^2+9\right)^{200}-\left(17-x^3\right)^{301}$.
a) Compute $f'(x)$. Explain why the following statement is correct: if $x>0$, then $f'(x)>0$.
b) Use calculus to explain why $f(76)>f(23)$. You must quote a specific result from this course and explain its relevance. Your answer to a) may be useful here.

(16) 5. A rectangular box with a square base is to be made from two different materials. The material for the top and four sides costs \$1 per square foot, while the material for the bottom costs \$2 per square foot. If you can spend \$196 on materials, what dimensions will maximize the volume of the box?

(10) 6. Find equations for all horizontal and vertical asymptotes of $f(x)=\frac{3+2e^{2x}}{5-7e^{2x}}$.

(10) 7. a) Give an example of a function which is not continuous. Explain why your example is not continuous.
b) Give an example of a function which is not differentiable. Explain why your example is not differentiable.

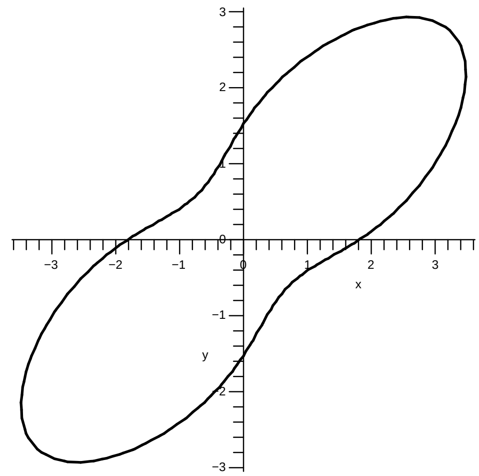
(12) 8. Suppose that y is implicitly defined as a function of x by the equation $x^4 - 24xy + 2y^4 = 11$.

a) Find $\dfrac{dy}{dx}$ in terms of y and x.

b) Find an equation for the line tangent to the graph at the point $P = (3, 1)$ which is on the graph.

Note There are some **Possibly useful numbers** on the formula sheet.

c) The program `Maple` gives the image shown to the right when asked to graph the equation. Sketch the tangent line found in b) on the image.



(18) 9. Suppose that $f'(x)$, the *derivative* of $f(x)$, is given by this formula:
$f'(x) = (x + 2)x^2(x - 1)^3$.

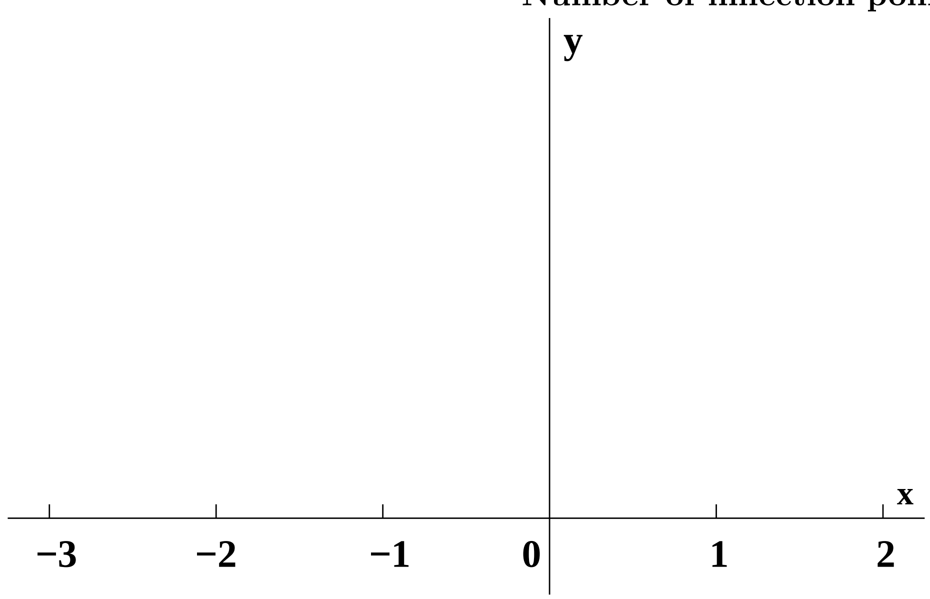
Note Read the formula carefully. Please <u>do not</u> try to find a formula for $f(x)$: this is not requested and will not help you answer any part of the problem.

a) What are the critical numbers of $f(x)$? For each critical number, explain why the associated critical point is a relative minimum, a relative maximum, or neither. Briefly support your answers using calculus.

b) Sketch a graph of $y = f(x)$ showing the features found in a) on the axes given. The graph should be as simple as possible. <u>Label each relative maximum</u> with **M** and <u>label each relative minimum</u> with **m**.

c) How many inflection points does your graph of $y = f(x)$ have? <u>Label each inflection point</u> with **I** on the graph drawn. Please do not compute $f''(x)$: <u>use the graph</u>.

Number of inflection points _______



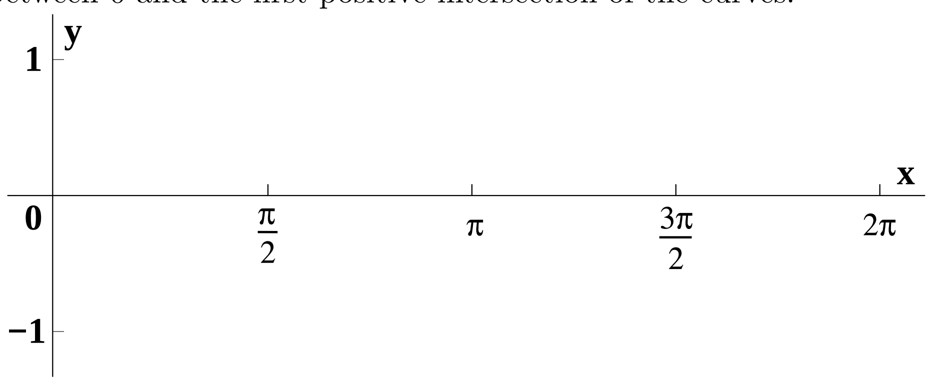
(12) 10. A spherical hot air balloon is being filled with air at the rate of 200 cubic feet per minute. At what rate is the radius of the balloon increasing when the balloon has 1000 cubic feet of air in it?

(12) 11. Suppose $f(x) = 3x^4 - 8x^3 - 18x^2 + 2$.
a) Find the exact maximum and minimum values of $f(x)$ for x in the interval $[-2, 2]$. Briefly explain your conclusions using calculus.
Note There are some **Possibly useful numbers** on the formula sheet.
b) If $[-2, 2]$ is the domain of the function $f(x)$, describe the range of $f(x)$ precisely. You may use your results in a) here. You must quote a specific result from this course and explain its relevance.

(8) 12. Compute the value of the Riemann sum for the function 3^x on the interval $[-1, 3]$ using the partition $\{-1, 0, 2, 3\}$ and taking as the sample points the right-hand endpoints of each subinterval.

(13) 13. Find the following indefinite integrals.
a) $\displaystyle\int \left(5\sqrt{x} - \frac{3}{x^2}\right) dx$
b) $\displaystyle\int e^{2x+x^2}(1+x)\, dx$
c) $\displaystyle\int \Big(\cos(x+3) + \sin(2x)\Big)\, dx$

(15) 14. Sketch both $y = \sin x$ and $y = \cos x$ for x in the interval $[0, 2\pi]$ on the axes given. Label the curves sketched. Compute the area bounded by the y-axis and the two curves when x is between 0 and the first positive intersection of the curves.



(12) 15. Find a formula for the solution, $x(t)$, of the initial value problem specified by the following information: $x''(t) = 2t - 3 + 4e^{-t}$ and $\begin{cases} x(0) = -1 \\ x'(0) = 5 \end{cases}$.

Final Exam for Math 135, section F2

August 16, 2006

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	20	
2	12	
3	22	
4	8	
5	16	
6	10	
7	10	
8	12	
9	18	
10	12	
11	12	
12	8	
13	13	
14	15	
15	12	
Total Points Earned:		