

Name _____ MA135 Final Exam A December 16, 2009

Instructor _____ Section _____

Be sure to show all of your work. All solutions should use calculus techniques from this course. Unsupported answers will receive no credit! Calculators are not allowed on this exam. You may only use the formula sheet and scratch paper supplied with this exam. Good Luck!!

Prob No.	Max Pts	Points	Prob No.	Max Pts	Points
1	18		8	17	
2	18		9	18	
3	18		10	18	
4	18		11	18	
5	17		12	18	
6	18		13	18	
7	18		14	18	
Subtotal	125		Subtotal	125	

Grand Total	
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1. (6 points each) Find the following limits, giving reasons for your answers. You may use any method from this course.

a. $\lim_{x \rightarrow 3} \frac{x - 3}{\sqrt{x} - \sqrt{3}} =$ _____

b. $\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{x^2} =$ _____

c. $\lim_{x \rightarrow \infty} x \sin\left(\frac{2}{x}\right) =$ _____

2. (9 points each) Find the derivatives of the following functions. You do not need to simplify your answers.

a. If $y = \cos^3 x \sin(x^5)$ then $\frac{dy}{dx} =$ _____

b. If $y = \frac{x \sin x}{1 + \ln x}$, then $\frac{dy}{dx} =$ _____

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3. (9 points each) Find the following indefinite integrals.

a. $\int \frac{t^2 + 5t + 1}{\sqrt{t}} dt = \underline{\hspace{4cm}}$

b. $\int \sin(\cos x) \sin x dx = \underline{\hspace{4cm}}$

4. (9 points each).

a. $\int_3^4 (1+e^x)^5 e^x dx =$ _____ Do not try to simplify your answer!

b. $\int_2^3 x\sqrt{x-1} dx =$ _____

5. (17 points) Let $f(x) = g(x^3 - 5)$. It is impossible to find $g(x)$, but a few values of $g(x)$ and $g'(x)$ are known: $g(1) = 2$, $g(2) = 5$, $g(3) = 7$, $g(4) = 2$, $g(5) = 11$, $g(6) = 13$, $g(7) = 21$, $g'(1) = 3$, $g'(2) = 2$, $g'(3) = 8$, $g'(4) = 10$, $g'(5) = 12$, $g'(6) = 21$ and $g'(7) = 23$.

a. Find $f(2)$. _____

b. find $f'(2)$. _____

6. (18 points) Find the equation of the tangent line to the curve described by

$$x^3y + xy^3 + x^2y - 2x^2 = -2$$

at the point $(1, 0)$. Any correct equation specifying this line is acceptable.

Tangent line:	
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7. (18 points) The tangent line to $y = f(x)$ at $x = 2$ is given by $y = 7x + 3$.
- (5 points) What is $f'(2)$? _____
 - (5 points) What is $f(2)$? _____
 - (6 points) Use linear approximation to approximate $f(2.1)$. _____
 - (2 points) If $f''(x) < 0$ for all x , is this approximation too large or too small? _____

8. (17 pts) Find the absolute maximum and minimum of the function $f(x) = x^3 + 3x^2 - 9x$ on the interval $[-2, 2]$.

Absolute max:	
Absolute min:	

9. (6 points each)

a. Find $\frac{dy}{dx}$ if $y = x^{8x}$.

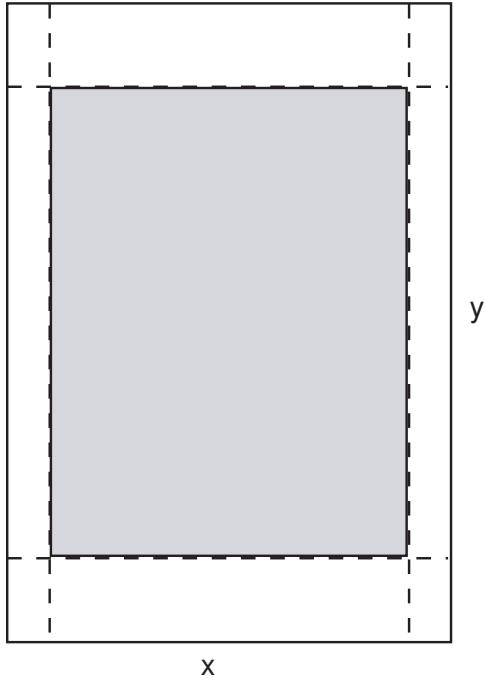
b. Find $\frac{dy}{dx}$ if $y = \int_0^x \sin t^2 dt$.

c. Find $\frac{dy}{dx}$ if $y = \int_0^{x^2} \sin t^2 dt$.

10. (18 points) A mad scientist sells radioactive bats to her friends. Experience tells her that she will sell 20 bats per month if she charges 30 dollars per bat and that each \$2 decrease in price will result in four more sales per month. How much should she charge per bat to maximize her revenue?

Price per bat:	
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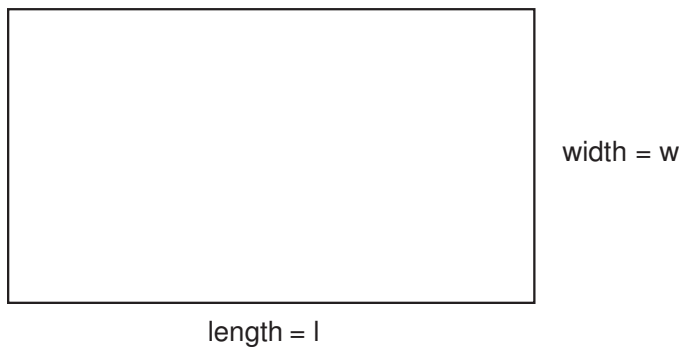
11. (18 points) A rectangular poster is to contain 80 square inches of printed matter with 5 inch margins at the top and bottom and 4 inch margins at the sides. If posterboard costs 10 cents per square inch, what are the dimensions of the least expensive poster satisfying the requirements?



12. (18 points) 5. (14 points) Compute the value of the Riemann sum for the function $f(x) = x^2$ on the interval $[1, 3]$ using $n = 4$ and taking x_k^* to be the midpoint of the k^{th} interval in the partition. You can leave your answer as a sum of fractions.

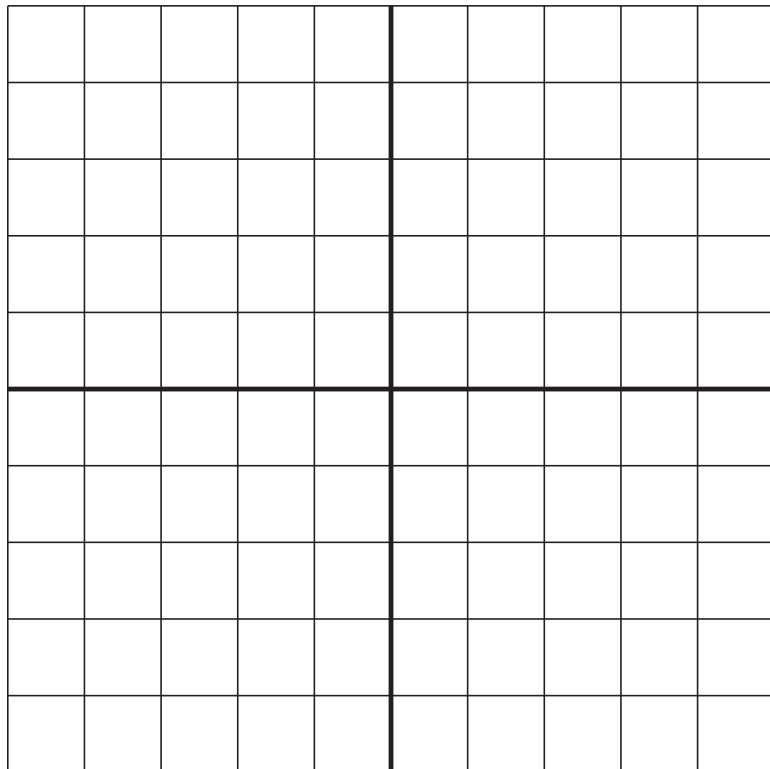
Value:	
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13. (18 points) The length of a rectangle is decreasing at 4 in/min and its width is increasing at 5 in/min. How fast is the length of the diagonal changing when the length is 8 in and the width is 6 in?



14. (18 points) Sketch the graph of the function $f(x) = \frac{x+2}{(x+1)^2}$. For this function,

$$f'(x) = -\frac{x+3}{(x+1)^3} \text{ and } f''(x) = \frac{2(x+4)}{(x+1)^4}.$$



Horizontal asymptote(s):	
Vertical asymptote(s):	
Increasing:	
Decreasing:	
Concave up:	
Concave down:	
Relative max/min:	
Inflections:	