Problems for the streaming review for exam 1

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I strongly suggest that you will do all the problem **before** watching.

1. Let

$$F(x) = \begin{cases} x^3 + A & x \leq 2 \\ 6x + 1 & 2 < x < 3 \\ x^2 + 2 & x \geq 3 \end{cases}$$

(a) Find the value of A that makes $F(x)$ continuous at $x = 2$.

(b) For the constant A from (a): is $F(x)$ differentiable at $x = 2$?

(c) Is $F(x)$ differentiable at $x = 3$?

2. Show that the equation $x^3 = x^2 + 1$ has at least one solution.

3. Use the definition of the derivative as a limit to find the derivative of $f(x) = \dfrac{1}{x+2}$

4. Find the center and the radius of the circle $x^2 + y^2 - 2x + 4y = 0$

5. Evaluate each of the following limits.

(a) $\lim\limits_{x\to 3} \dfrac{\sqrt{x} - \sqrt{3}}{x - 3} + 3x^2$ (b) $\lim\limits_{x\to 0} \dfrac{\tan(5x)}{\sin(3x)}$ (c) $\lim\limits_{x\to 3} \dfrac{|x-3|}{x^2 - 9}$

6. Find the derivatives of the following functions.

(a) $F(x) = \sqrt{\dfrac{e^x}{x^2+3}}$ (b) $g(x) = 3\cos^4 x \cdot \sin x^9$ (c) $f(x) = e^{\sin x}$

(d) $h(x) = \left(\dfrac{x^2 - \ln x}{3x+2}\right)^9$ (e) $F(x) = \dfrac{\tan x(x^2 - 4x)}{\ln x}$

7. Let $g(x)$ be a differentiable function such that $g(1) = 2,\ g(2) = 5,\ g(3) = 7,$ $g(4) = 2,\ g'(1) = 3,\ g'(2) = 2,\ g'(3) = 8,\ g'(4) = 10$. Let $f(x) = x^2 + x$.

Find the exact value of: (a) $(gf)'(2)$ (b) $\left(\dfrac{f}{g}\right)'(3)$ (c) $(g \circ f)(1)$.

8. Solve $\log(5x) + \log(x-1) - 2 = 0$

9. (a) Describe the rectilinear motion given at time t (in seconds) by $s(t) = 20 + 8t - t^2$ (in meters), for $0 \leq t \leq 10$.

(b) Find the total distance traveled.

10. Find the domain of the function $f(x) = \ln(x^2 - 4)$