Name ______________________________ MA135 Final Exam A May 9, 2013

Instructor __________________________ Section _____

Be sure to show all of your work. All solutions should use calculus techniques from this course. Unsupported answers will receive no credit! Calculators are not allowed on this exam. You may only use the formula sheet and scratch paper supplied with this exam. Good Luck!!

Prob No.	**Max Pts**	**Points**	**Prob No.**	**Max Pts**	**Points**
1	18		8	17	
2	18		9	18	
3	18		10	18	
4	18		11	18	
5	17		12	18	
6	18		13	18	
7	18		14	18	
Subtotal	125		**Subtotal**	125	

Grand Total	

1. (9 points each) Find the derivatives of the following functions. You do not need to simplify your answers.

a. If $y = (x^3 + x)^{10}$ then $\dfrac{dy}{dx} =$ ____________________

b. If $y = \ln(x^8)$, then $\dfrac{dy}{dx} =$ ____________________

2. (9 points each) Find the following indefinite integrals.

a. $\int \frac{x+1}{x}\, dx =$ ____________________

b. $\int (2x+3)^{10}\, dx =$ ____________________

3. (9 points each) Find the following definite integrals.

a. $\displaystyle\int_0^1 e^x(1+e^{-2x})\,dx =$ ____________________

Hint: Multiply it out.

b. $\displaystyle\int_0^{\pi/2} (1+\sin(x))^5\cos(x)\,dx =$ ________________________________

4. (18 points) If $x^3 + xy + y^2 = 7$, find $\frac{dy}{dx}$ at $(1, 2)$. ________________________________

5. (17 points) Prove that the equation $\frac{1}{x+1} = x^2 - x - 1$ has at least one solution on the interval $(1, 2)$.

6a. (9 points) Find $\lim_{x \to 0} \frac{\sin^2 x}{\sin(2x^2)}$.

Answer	

6b. (9 points) Find $\lim_{x \to 1} \frac{\ln(x^2 + 2) - \ln(3)}{x - 1}$.

Answer	

7. (18 pts) A peach grower has determined that if 30 trees are planted per acre, each tree will average 200 lb of peaches per season. However, for each tree grown in addition to the 30 trees, the average yield for each of the trees in the grove drops by 5 lb per tree. How many trees should be planted on each acre to maximize the number of pounds of peaches harvested per acre?

Answer	

8. (17 pts) A car traveling north at 40 mi/hr and a truck traveling east at 30 mi/hr leave an intersection at the same time. At what rate will the distance between them be changing 4 hours later?

Answer:	

9. (18 points) Find the absolute maximum and minimum of the function

$$f(x) = (6x + 1)e^{3x}$$

on the interval $[-1000, 1000]$. Hint: You can figure this out without a calculator if you use the first derivative test and think about the signs at the endpoints.

maximum:	
minimum:	

10. (18 points) The marginal revenue of a certain commodity is

$$R'(x) = -3x^2 + 4x + 32$$

where x is the level of production in thousands. Assume $R(0) = 0$. Find $R(x)$. What is the demand function $p(x)$? Find the level of production that maximizes revenue. You do not need to check that your answer is a maximum.

R(x):	
p(x):	
x that maximizes revenue:	

11. (9 points each)

a. If $F(x) = \int_0^{x^3+x} \frac{t}{1+t^6}\, dt$, find $F'(x)$.

F'(x)=	

b. If $f(x) = \begin{cases} x & x < 1 \\ \frac{1}{x} & x \geq 1 \end{cases}$ find $\int_0^5 f(t)\, dt$.

$\int_0^5 f(t)\, dt=$	

12. (18 points) Use linear approximation or differentials to approximate $(8.02)^{1/3}$.

Approx value of $(8.02)^{1/3}$**:**	

13. (6 points apiece) A manufacturer of car batteries estimates that the fraction of his batteries will work for at least t months is

$$p(t) = e^{-0.03t}.$$

a. What fraction of the batteries can be expected to last at least 40 months?
b. What fraction can be expected to fail before 50 months?
c. What fraction can be expected to fail between the 40th and 50th months?

Last 40 months:	
Fail before 50 months:	
Fail between 40 and 50 months:	

14. (18 points) Consider the function $f(x) = \dfrac{(x-1)^2}{(x+2)(x-4)}$. For this function, we have $f'(x) = \dfrac{-18(x-1)}{(x+2)^2(x-4)^2}$ and $f''(x) = \dfrac{54((x-1)^2+3)}{(x+2)^3(x-4)^3}$.

Horizontal asymptotes:	
Find vertical asymptotes:	
The function is increasing on the interval(s):	
The function is decreasing on the interval(s):	
The function is concave up on the interval(s):	
The function is concave down on the interval(s):	
There are inflection(s) at:	
There are local maxima at:	
There are local minima at:	

Sketch the graph below