Answers to Review Problems for Calculus 135 Exam II

Short answers are listed first. Detailed answers are given following the list of short answers.

Short Answers

1. See detailed answers.

2 a.: There is a relative minimum of -51 at $x = 3$ and a relative maximum of 13 at $x = 1$.

2 b.: The absolute minimum on $[-2, 2]$ is -41. The absolute maximum on $[-2, 2]$ is 13.

3.: x = $\sqrt[3]{40}$ inches and $y = \frac{100}{\sqrt[3]{1600}}$ inches.

4. See detailed answers.

5. See detailed answers.

6.:

$$y' = \left(4\left(\frac{5x^4 + 12x}{x^5 + 6x^2}\right) + \frac{6x^5 + 4x^3}{x^6 + x^4 + 12} - 2\left(\frac{2x}{x^2 + 5}\right) - \frac{8}{x}\right)\left(\frac{(x^5 + 6x^2)^4(x^6 + x^4 + 12)}{(x^2 + 5)^2 x^8}\right)$$

7.: $x = e^{\frac{32}{5}}$

8 a.: $x = 0$ and $x = -1$ are the critical points. $f(x)$ is decreasing on $(-\infty, -1)$ and $(-1, 0)$ and increasing on $(0, +\infty)$.

8 b.: $f''(x) = 5x^{14}(x^4 + 1)^9(5x + 3)$. The points of inflection occur at $x = -1$ and $x = -3/5$.

9. See detailed answers.

10.: The absolute maximum is $\sqrt{2}$. The absolute minimum $-\sqrt{2}$.

11.: $\sqrt[3]{8.01} \approx 2 + \frac{1}{1200}$ and $\sqrt[3]{7.99} \approx 2 - \frac{1}{1200}$

12.: The unit price is decreasing by approximately \$ 8 per chair demanded per week.

13. See detailed answers.

14.:

a. The only critical point is $x = -2.7$ The only relative extrema is the relative minimum $f(-2.7)$.

b. $f(x)$ is decreasing on $(-\infty, -2.7)$ and $f(x)$ is increasing on $(-2.7, +\infty)$.

c. The inflection points are $(-1, f(-1))$ and $(1.5, f(1.5))$. The function $f(x)$ is concave up on $(-\infty, -1)$ and on $(1.5, +\infty)$. The function $f(x)$ is concave down on $(-1, 1.5)$

Detailed Answers

1 ANSWER: The description of a graph which is a solution is given below:

Since $f(x)$ is **continuous for all** x, there are no holes, jumps, breaks or vertical asymptotes in the graph. Since $f(x)$ is **differentiable of all** x, in addition to there being no holes, jumps, breaks or vertical asymptotes in the graph, there are no sharp corners in the graph or points at which the graph has a vertical tangent. Note: Since a function that is differentiable for all x, is automatically continuous for all x, the above guidelines would be the same even if the instructions had simply read "Sketch a graph of a differentiable function $f(x)$ with the following properties ... etc."

Since $\lim\limits_{x\to+\infty} = -\infty$ and $\lim\limits_{x\to-\infty} = -\infty$ there are no horizontal asymptotes in the graph.

Now, there are no values of x for which $f'(x)$ is undefined, and no values of x for which $f(x)$ is undefined so the fact that $x = 0$, $x = 2$ and $x = 6$ are the only points at which $f'(x) = 0$ means that the only critical points of $f(x)$ are $x = 0$, $x = 2$ and $x = 6$. Our graph has a horizontal tangent line only at $x = 0$, $x = 2$, and $x = 6$. The graph is increasing on $(-\infty, 0)$, and $(2, 6)$, and is decreasing on $(0, 2)$ and $(6, +\infty)$. The graph is concave up on $(1, 4)$ and concave down on $(-\infty, 1)$ and on $(4, +\infty)$ The graph has inflection points at $x = 1$ and at $x = 4$.

2 a. ANSWER: There is a relative minimum of -51 at $x = 3$ and a relative maximum of 13 at $x = 1$.

2 a. IN DETAIL:

To find the extrema of a function, we begin by finding the first derivative of the function: $f'(x) = 6x^2 - 12x - 18 = 6(x - 3)(x + 1)$.
We then solve for those values of x in the domain of $f(x)$ for which the first derivative is undefined or equal to zero (critical points). The critical points are $x = 3$, and $x = -1$.

To use the first derivative test to determine which critical points, if any, give relative maxima or relative minima, we test the sign of $f'(x)$ at a sample point in each of the intervals determined by the critical points.

Testing the sign of $f'(x)$ at a sample point in the interval $(\infty, -1)$ we find that $f'(x) > 0$ on $(-\infty, -1)$.
Testing the sign of $f'(x)$ at a sample point in the interval $(-1, 3)$ we find that $f'(x) < 0$ on $(-1, 3)$.
Testing the sign of $f'(x)$ at a sample point in the interval $(3, +\infty)$ we find that $f'(x) > 0$ on $(3, +\infty)$.
Since $f'(x)$ changes from $''+''$ on the left of -1, to $''-''$ on the right of -1, $f(x)$ attains a relative maximum at $x = -1$.
Since $f'(x)$ changes from $''-''$ on the left of 3, to $''+''$ on the right of 3, $f(x)$ attains a relative minimum at $x = 3$.

To find the actual relative maximum value, we evaluate $f(x)$ at $x = -1$. $f(-1) = 13$. Thus $f(-1) = 13$ is a relative maximum of $f(x)$.

To find the actual relative minimum value of $f(x)$ we evaluate $f(x)$ at $x = 3$. $f(3) = -51$. Thus the relative minimum of $f(x)$ is $f(3) = -51$.

There are no other relative maxima or minima.

If we wanted to use the second derivative test to determine whether a relative maximum or relative minimum occurred at $x = -1$ and $x = 3$, we would take the second derivative: $f''(x) = 12x - 12$. We test the sign of $f''(x)$ at each of our critical numbers, -1 and 3. Since $f''(-1) = -24$ which is less than zero, $f(-1) = 13$ is a relative maximum. Since $f''(3) = 24$ which is greater than zero, $f(3) = -51$ is a relative minimum.

2 b. ANSWER: The absolute maximum on $[-2, 2]$ is 13. The absolute minimum on $[-2, 2]$ is -41.

2 b. IN DETAIL:

To find the absolute maximum and minimum of $f(x)$ on the closed interval $[-2, 2]$ we would begin as in part (a) above to find the critical points of $f(x)$, except we restrict our attention to those critical points in the closed interval $[-2, 2]$. We evaluate $f(x)$ at each of the endpoints of the closed interval, as well as at those critical points that fall in the closed interval. Since $x = 3$ is not in this closed interval, we do not include it in our analysis:
$f(-2) = -1, \quad f(2) = -41, \quad$ and $\quad f(-1) = 13$.
The largest of these values is 13, and so 13 is the absolute maximum value of $f(x)$ on the closed interval $[-2, 2]$ The smallest of these values is -41, so -41 is the absolute minimum value of $f(x)$ on the closed interval [-2,2].

3. ANSWER: x = $\sqrt[3]{40}$ inches and $y = \frac{100}{\sqrt[3]{1600}}$ inches.

3. IN DETAIL:

Since the base of the box is a square, we can use x for both the length and the width of the base in inches. We can let y be the height of the box in inches and C be the cost of the box in dollars. Now, we are given that the volume of the box must be 100 cubic inches. This gives us a relationship between x and y, namely: $100 = x^2y$

We want to determine the dimensions, x and y, that will minimize C which is the cost of the box in dollars. To do this we begin by determining an expression for C as a function of the dimensions x and y. We are given that the cost of the base is 4 dollars per square inch. The are x^2 square inches in the base so the cost of the base in dollars is $4x^2$. The cost of the sides is 80 cents per square inch. There are xy square inches in each side, so the cost in dollars of each side is $0.80xy$. Since the are 4 sides, one base and no top, the cost of the box in dollars as a function of the dimensions x and y is given by:

$$C = 4x^2 + 3.2xy$$

Now, in order to apply our techniques for finding the minimum value for our cost function, we need to express our cost as a function of **one** variable. To do this we use the relationship we were given between x and y, namely that $y = \frac{100}{x^2}$. Using this gives us the cost C as a function of x:

$$C(x) = 4x^2 + 3.2x\frac{100}{x^2}$$

or

$$C(x) = 4x^2 + \frac{320}{x}$$

(Note, we could have chosen to write cost as a function of y.)

Next we need to determine the domain of this function $C(x)$. The domain of $C(x)$ is $(0, +\infty)$. Now we take the first derivative of this cost function:

$$C'(x) = 8x - \frac{320}{x^2}$$

Solving for the values of x for which $C'(x) = 0$, we get $x^3 = 40$ or $x = \sqrt[3]{40}$. Since this value of x is in the domain of the cost function, we can use it as a critical point. Now to find y, we use the relationship $100 = x^2y$ which gives us $y = \frac{100}{\sqrt[3]{40^2}} = \frac{100}{\sqrt[3]{1600}}$.

To verify that our cost is, in fact, minimized (as opposed to maximized) when $x = \sqrt[3]{40}$ we can use the first or second derivative test.

$C'(x)$ changes from "–" for values of x to the left of $\sqrt[3]{40}$ to "+" for values of x to the right of $\sqrt[3]{40}$. So, by the first derivative test, $C(x)$ is minimized when $x = \sqrt[3]{40}$.

If you prefer to use the second derivative test we have: $C''(\sqrt[3]{40})$ is positive, so by the second derivative test, $C(x)$ is minimized when $x = \sqrt[3]{40}$.

Both the first and second derivative test are only tests for relative extrema. To show that C in fact has an absolute minimum at $x = \sqrt[3]{40}$, you need to say something more: either that that there is only the one critical point of C (if there were two or more critical points, then a local test would not be sufficient), or that C is concave up in the WHOLE domain, or that C' is negative for ALL x in the domain less than $\sqrt[3]{40}$ and positive for all x in the domain greater than $\sqrt[3]{40}$.

4. ANSWER: The domain of $f(x) = \frac{x^2-1}{x^2-4}$ is: $(-\infty, -2) \bigcup (-2, 2) \bigcup (2, +\infty)$

$f(0) = \frac{1}{4}$

The vertical asymptotes are vertical lines of the form $x = a$ where the values of "a" are those values of x for which:

$\lim_{x \to a^+} f(x) = +\infty$, or $\lim_{x \to a^-} f(x) = +\infty$, or
$\lim_{x \to a^+} f(x) = -\infty$, or $\lim_{x \to a^-} f(x) = -\infty$.

For this function, the values of 2 and -2 are the only two such values of "a". To see this:

$\lim_{x \to -2^+} f(x) = -\infty$, and $\lim_{x \to -2^-} f(x) = +\infty$, and
$\lim_{x \to 2^+} f(x) = +\infty$, and $\lim_{x \to 2^-} f(x) = -\infty$.

Thus the vertical asymptotes are the vertical lines $x = 2$ and $x = -2$.

The horizontal asymptotes are horizontal lines of the form $y = b$ where the values of "b" are those values of y for which:

$\lim_{x \to +\infty} f(x) = b$, or $\lim_{x \to -\infty} f(x) = b$.

For this function: $\lim_{x \to +\infty} f(x) = 1$, and $\lim_{x \to -\infty} f(x) = 1$.
Thus the horizontal asymptote is the horizontal line $y = 1$.

To find the extema, and the intervals over which $f(x)$ is increasing and decreasing, we look at the first derivative. Using the quotient rule, and simplifying the numerator, we get:

$$f'(x) = \frac{(x^2-4)(2x) - (x^2-1)(2x)}{(x^2-4)^2} = \frac{-6x}{(x^2-4)^2}$$

Our only critical point is $x = 0$. Remember, even though the first derivative is undefined for $x = \pm 2$, we do not use $x = \pm 2$ as critical points, because the numbers 2 and -2 are not in the domain of $f(x)$. Even though 2 and -2 are not in the domain of $f(x)$, we can use them along with 0 to determine the intervals in which we will test the sign of $f'(x)$ to see on which intervals $f(x)$ is increasing or decreasing.

Now, $f'(x)$ is positive on the intervals $(-\infty, -2)$ and $(-2, 0)$. $f'(x)$ is negative on the intervals $(0, 2)$ and $(2, +\infty)$.

This tells us that the function $f(x)$ is increasing on the intervals $(-\infty, -2)$ and $(-2, 0)$. It also tells us that the function $f(x)$ is decreasing on the intervals $(0, 2)$ and $(2, +\infty)$. Also, by the first derivative test, we know that $f(x)$ has a relative maximum at $x = 0$. That relative maximum is $f(0) = \frac{1}{4}$.

To find the points of inflection, and the intervals over which the graph of the function $f(x)$ is concave up and concave down, we look at the second derivative.

We use the quotient rule on the first derivative and simplify to find:

$$f''(x) = \frac{-6(x^2-4)^2 - [-6x(2)(x^2-4)(2x)]}{(x^2-4)^4} = \frac{-6(x^2-4) - [-6x(2)(2x)]}{(x^2-4)^3}$$

$$= \frac{18x^2+24}{(x^2-4)^3}$$

The second derivative does not equal zero at any values of x, but it is undefined at the values -2 and 2. Even though these points are not in the domain of $f(x)$ we can use them to determine the intervals in which we will test the sign of $f''(x)$. Testing the sign of $f''(x)$ at sample points in the intervals $(-\infty, -2)$, $(-2, 2)$ and $(2, +\infty)$, we find that $f''(x)$ is positive on the intervals $(-\infty, -2)$ and $(2, +\infty)$, and negative on $(-2, 2)$. This means that $f(x)$ is concave up on the intervals $(-\infty, -2)$ and $(2, +\infty)$, and concave down on $(-2, 2)$. Now, ordinarily we would claim: since concavity changed about the points $x = -2$ and $x = 2$ we have inflection points at $x = -2$ and $x = 2$. **HOWEVER, since neither -2 nor 2 are in the domain of $f(x)$, and there are no other possible inflections points, there are NO inflection points for the graph of $f(x)$.**

5. ANSWER: The top row of numbers in the table below are the values of x. The following three rows of signs are the signs of the functions $G(x)$, $G'(x)$, and $G''(x)$ at these values of x.

	$-.05$	0	0.5	1	2	2.5	3
G	$-$	$+$	$+$	$+$	$-$	$-$	$-$
G'	$+$	$+$	$+$	0	$-$	0	$+$
G''	$-$	$-$	$-$	$-$	$+$	$+$	$+$

5. IN DETAIL: The function $G(x)$ will be positive for those values of x for which the graph of $G(x)$ is above the x axis. The function $G(x)$ will be zero for those values of x at which the graph of $G(x)$ crosses the x axis. The function $G(x)$ will be negative for those values of x for which the graph of $G(x)$ is below the x axis.

The function $G'(x)$ is positive for those values of x for which the function $G(x)$ is increasing, i.e., for those values of x for which the graph of $G(x)$ is "rising" (tangent lines have positive slope).

The function $G'(x)$ is negative for those values of x for which the function $G(x)$ is decreasing, i.e., for those values of x for which the graph of $G(x)$ is "falling" (tangent lines have negative slope).

The function $G'(x)$ is zero for those values of x for which graph of the function $G(x)$ has a horizontal tangent line (tangent line has a slope equal to zero).

The function $G''(x)$ is positive for those values of x for which the graph of $G(x)$ is concave up (like a cup). The function $G''(x)$ is negative for those values of x for which the graph of $G(x)$ is concave down (like a frown).

6. ANSWER:

$$y' = \left(4\left(\frac{5x^4+12x}{x^5+6x^2}\right) + \frac{6x^5+4x^3}{x^6+x^4+12} - 2\left(\frac{2x}{x^2+5}\right) - \frac{8}{x}\right)\left(\frac{(x^5+6x^2)^4(x^6+x^4+12)}{(x^2+5)^2x^8}\right)$$

6. IN DETAIL: We begin by writing

$$y = \frac{(x^5+6x^2)^4(x^6+x^4+12)}{(x^2+5)^2x^8}$$

Next we take the natural log of both sides:

$$\ln y = \ln\frac{(x^5+6x^2)^4(x^6+x^4+12)}{(x^2+5)^2x^8}$$

We use properties of logs to simplify the expression to get:

$$\ln y = \ln{(x^5+6x^2)}^4 + \ln(x^6+x^4+12) - \ln{(x^2+5)}^2 - \ln x^8$$

Using properties of logs to simplify further we get:

$$\ln y = 4\ln(x^5+6x^2) + \ln(x^6+x^4+12) - 2\ln(x^2+5) - 8\ln x$$

Next we use implicit differentiation to get:

$$\frac{1}{y}y' = 4\left(\frac{5x^4+12x}{x^5+6x^2}\right) + \frac{6x^5+4x^3}{x^6+x^4+12} - 2\left(\frac{2x}{x^2+5}\right) - \frac{8}{x}$$

Then we multiply both sides of the equation by y. Then we substitute the original formula for y to get:

$$y' = \left(4\left(\frac{5x^4+12x}{x^5+6x^2}\right) + \frac{6x^5+4x^3}{x^6+x^4+12} - 2\left(\frac{2x}{x^2+5}\right) - \frac{8}{x}\right)\left(\frac{(x^5+6x^2)^4(x^6+x^4+12)}{(x^2+5)^2x^8}\right)$$

7. ANSWER: $x = e^{\frac{32}{5}}$

7. IN DETAIL:

Using properties of logs and exponentials we get:

$$\ln x^2 + \ln x^3 = 32$$

Continuing to use properties of logs we get: $\ln x^2 + \ln x^3 = \ln x^2 x^3 = \ln x^5 = 5 \ln x$ and so we get: $5 \ln x = 32$ which gives us: $\ln x = \frac{32}{5}$ which gives:

$$x = e^{\frac{32}{5}}$$

8 a. ANSWER: $x = 0$ and $x = -1$ are the critical points. $f(x)$ is decreasing on $(-\infty, -1)$ and $(-1, 0)$ and increasing on $(0, +\infty)$.

8. a. IN DETAIL:

The values of x in the domain of $f(x)$ for which the first derivative is zero or undefined are the critical points. The first derivative, which is given, is defined for all x since it is a polynomial; it is zero at $x = 0$ and at $x = -1$. $f(x)$ is increasing on those intervals in which the first derivative is positive. $f(x)$ is decreasing on those intervals in which the first derivative is negative. Since the first derivative is zero at $x = 0$ and at $x = -1$ we test the sign of the first derivative in the intervals $(-\infty, -1)$, $(-1, 0)$ and $(0, +\infty)$. We test the sign of $f'(x)$ by evaluating it at sample points in each of these intervals. The first derivative is negative on $(-\infty, -1)$ and $(-1, 0)$. The first derivative is positive on $(0, +\infty)$. This means that $f(x)$ is decreasing on $(-\infty, -1)$ and $(-1, 0)$ and increasing on $(0, +\infty)$.

8. b. ANSWER: $f''(x) = 5x^{14}(x^4 + 1)^9(5x + 3)$. The points of inflection occur at $x = -1$ and $x = -3/5$.

8. b. IN DETAIL: We use the product rule on $f'(x)$ to get:

$$f''(x) = 10(x + 1)^9 x^{15} + 15x^{14}(x + 1)^{10} = x^{14}(x + 1)^9(10x + 15(x + 1))$$

$$= x^{14}(x + 1)^9(25x + 15) = 5x^{14}(x + 1)^9(5x + 3)$$

The points at which $f''(x)$ is zero are $x = -1$, $x = -\frac{3}{5}$ and $x = 0$. To test which of these are, in fact, inflection points we test the sign of $f''(x)$ at a sample point in each of the intervals $(-\infty, -1)$, $(-1, -\frac{3}{5})$, $(-\frac{3}{5}, 0)$ and $(0, +\infty)$ We find that $f''(x)$ is positive on $(-\infty, -1)$, $(-\frac{3}{5}, 0)$ and $(0, +\infty)$ and negative on $(-1, -\frac{3}{5})$. Now, the second derivative changes sign from positive on the left of $x = -1$ to negative on the right of $x = -1$ and from negative on the left of $x = -\frac{3}{5}$ to positive on the right of $x = -\frac{3}{5}$. This means that -1 and $-\frac{3}{5}$ are the x coordinates of the inflection points. There is no inflection point at $x = 0$ since the second derivative does not change sign about the value $x = 0$.

9. ANSWER:

$$F'(x) = 2 \tan x \sec^2 x + 8e^x \cos 8x + e^x \sin 8x$$

If you used logarithmic differentiation, your answer for $G'(x)$ would look like:

$$G'(x) = \frac{2x}{x^2+5} - \frac{1}{x+6} + 3x^2 e^{x^3}$$

otherwise the answer would look like:

$$G'(x) = \left(\frac{1}{\frac{x^2+5}{x+6}}\right)\left(\frac{(x+6)(2x)-(x^2+5)}{(x+6)^2}\right) + 3x^2 e^{x^3}$$

Note: this can be simplified to look the same as the answer you would have gotten if you used logarithmic differentiation.

$$H'(x) = -2x\sin x^2 + 0 + x^2e^x + 2xe^x + 0$$

Note: $\cos x^2$ is understood to mean $\cos(x^2)$ whereas $\cos^2 x$ is understood to mean $(\cos x)^2$.

9. IN DETAIL:

Note: $\tan^2 x$ is understood to mean $(\tan x)^2$ whereas $\tan x^2$ would be understood to mean $\tan(x^2)$. To find $F'(x)$ we use the chain rule on $\tan^2 x$ and the product rule on the product e^x times $\sin 8x$. The chain rule is used on $\sin 8x$ as shown below:

$$\begin{aligned} F'(x) &= (2\tan x)(\frac{d}{dx}\tan x) + (e^x)(\frac{d}{dx}\sin 8x) + (\frac{d}{dx}e^x)\sin 8x \\ &= 2\tan x\sec^2 x + 8e^x\cos 8x + e^x\sin 8x \end{aligned}$$

10. ANSWER: The absolute maximum is $\sqrt{2}$. The absolute minimum $-\sqrt{2}$.

10. IN DETAIL:

We use the same approach to find the absolute maximum and absolute minimum on $[0, 2\pi]$ as we used in problem 2 to find the absolute maximum and absolute minimum on $[-2, 2]$. We begin by finding the first derivative:

$$g'(x) = \cos x - \sin x$$

Since both $g(x)$ and $g'(x)$ are defined for all x, in particular for all x in domain $[0, 2\pi]$ the critical points are those values of x in $[0, 2\pi]$ for which $g'(x)$ will equal 0, i.e. those points in $[0, 2\pi]$ for which $\sin x = \cos x$. These values are $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$. If you remember your precalculus, $\cos x$ and $\sin x$ have the same value for a 45° angle. This means they would also have the same value at a 225° angle. There are many angles for which $\sin x$ and $\cos x$ have the same value, but remember that we are only interested in angles in the interval $[0, 2\pi]$, that is, $[0, 360°]$. In calculus we typically use radian measure; and so 45° is $\frac{\pi}{4}$ radians and 225° is $\frac{5\pi}{4}$ radians.

Next we evaluate $g(x)$ at the endpoints 0 and 2π as well as at the critical points $\frac{\pi}{4}$ and $\frac{5\pi}{4}$:

$$g(0) = 0 + 1 = 1 \qquad g(2\pi) = 0 + 1 = 1$$

$$g(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2} \qquad g(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$$

Since $\sqrt{2}$ is the largest of these values, the absolute maximum of $g(x)$ on $[0, 2\pi]$ is $\sqrt{2}$. Since $-\sqrt{2}$ is the smallest of these values, the absolute minimum of $g(x)$ on $[0, 2\pi]$ is $-\sqrt{2}$.

11. ANSWER: $\sqrt[3]{8.01} \approx 2 + \frac{1}{1200}$ and $\sqrt[3]{7.99} \approx 2 - \frac{1}{1200}$

11. IN DETAIL:

$$\sqrt[3]{8.01} \approx \sqrt[3]{8} + f'(8)dx$$

where dx is .01 since 8.01 is .01 greater than 8.
Since $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$, $f'(x) = \frac{1}{3x^{\frac{2}{3}}} = \frac{1}{3\sqrt[3]{x^2}}$ which gives us:

$$\sqrt[3]{8.01} \approx \sqrt[3]{8} + \frac{1}{3\sqrt[3]{8^2}}(.01)$$

and so we get $\sqrt[3]{8.01} \approx 2 + \frac{1}{1200}$. Now,

$$\sqrt[3]{7.99} \approx \sqrt[3]{8} + f'(8)dx$$

where dx is $-.01$ since 7.99 is .01 less than 8.
Since $f(x) = \sqrt[3]{x}$, $f'(x) = \frac{1}{3\sqrt[3]{x^2}}$ which gives us:

$$\sqrt[3]{7.99} \approx \sqrt[3]{8} + \frac{1}{3\sqrt[3]{8^2}}(-.01)$$

and so we get $\sqrt[3]{7.99} \approx 2 - \frac{1}{1200}$.

12. ANSWER: The unit price is decreasing by approximately \$ 8 per chair demanded per week.

12. IN DETAIL:

The price function can be written as $p = 280 - 0.2x^2$. The change in price of a chair when the weekly quantity demanded changes from 20 to 21 is approximated by the differential dp. Using our formula for differentials: $dp = p'(x)dx$, we get:

$$dp = -0.4xdx$$

where $dx = 1$ since there is an increase of 1 chair demanded per week. Our original value of x was 20 so we get:

$$dp = -0.4(20)(1) = -8$$

Thus the price is changing at a negative rate, that is, it is deceasing, and so the unit price is decreasing by approximately \$ 8 per chair demanded per week.

13. ANSWER: The function $F(x) = x^2 + 1$ is a polynomial, so it is continuous on the closed interval $[0, 4]$ and differentiable on the open interval $(0, 4)$. Since it satisfies the hypothesis of the Mean Value Theorem (that is, the "if" part of the theorem), we can apply the conclusion of the Mean Value Theorem (that is, the "then" part of the theorem).

The conclusion of the Mean Value Theorem says that there is a number c in the interval $(0, 4)$ such that

$$f'(c) = \frac{f(4) - f(0)}{4 - 0} = \frac{17 - 1}{4} = 4$$

Now, $f'(x) = 2x$ and $f'(c) = 2c$. $f'(c) = 2c$ will be equal to 4 when $c = 2$. If you look at your graph, you should see that the slope of the tangent line to the curve $x^2 + 1$ at $x = 2$ is 4.

14. ANSWER:

a. The only critical point is $x = -2.7$ The only relative extrema is the relative minimum $f(-2.7)$.

b. $f(x)$ is decreasing on $(-\infty, -2.7)$ and $f(x)$ is increasing on $(-2.7, +\infty)$.

c. The inflection points are $(-1, f(-1))$ and $(1.5, f(1.5))$. The function $f(x)$ is concave up on $(-\infty, -1)$ and on $(1.5, +\infty)$. The function $f(x)$ is concave down on $(-1, 1.5)$

14. IN DETAIL:

a. Since this is a graph of $f'(x)$ and the critical numbers are those values of x in the domain of $f(x)$ for which $f'(x)$ equals zero or is undefined, the only critical number is $x = -2.7$. (At $x = -2.7$ the graph of $f'(x)$ crosses the x axis so $f'(-2.7) = 0$). [Note: We know that -2.7 is in the domain of $f(x)$ because $f(x)$ has a derivative at -2.7]. Since the graph of $f'(x)$ is below the x axis to the left of -2.7 and above the x axis to the right of -2.7, we know that $f'(x)$ is negative to the left of -2.7 and positive to the right of -2.7. Thus, since $f'(x)$ changes from "–" on the left of -2.7 to "+" on the right of -2.7, by the first derivative test, $f(-2.7)$ is a relative minimum of $f(x)$.

b. The function $f(x)$ will be increasing everywhere $f'(x)$ is positive (i.e., everywhere the graph of $f'(x)$ is above the x axis). The function $f(x)$ will be decreasing every where $f'(x)$ is negative, (i.e., everywhere the graph of $f'(x)$ is below the x axis). This means that $f(x)$ is decreasing on $(-\infty, -2.7)$ and $f(x)$ is increasing on $(-2.7, +\infty)$.

c. Even though we don't have a graph of $f''(x)$ we know that the function $f(x)$ will be concave up everywhere the second derivative $f''(x)$ is positive. The second derivative $f''(x)$ will be positive everywhere the first derivative $f'(x)$ is increasing. This means that the function $f(x)$ is concave up on $(-\infty, -1)$ and on $(1.5, +\infty)$.

We know that the function $f(x)$ will be concave down everywhere its second derivative $f''(x)$ is negative. The second derivative $f''(x)$ will be negative everywhere the first derivative $f'(x)$ is decreasing. This means that the function $f(x)$ is concave down on $(-1, 1.5)$.

The points at which the function $f(x)$ changes concavity are the inflection points. Changes in concavity occur about the points with x coordinates $x = -1$ and $x = 1.5$. Since we do not have the graph of $f(x)$ and do not know the formula for $f(x)$ we cannot determine the y coordinates of these points. So we simply write that the points of inflection are $(-1, f(-1))$ and $(1.5, f(1.5))$.